Bayesian Nonparametric Spectral Estimation

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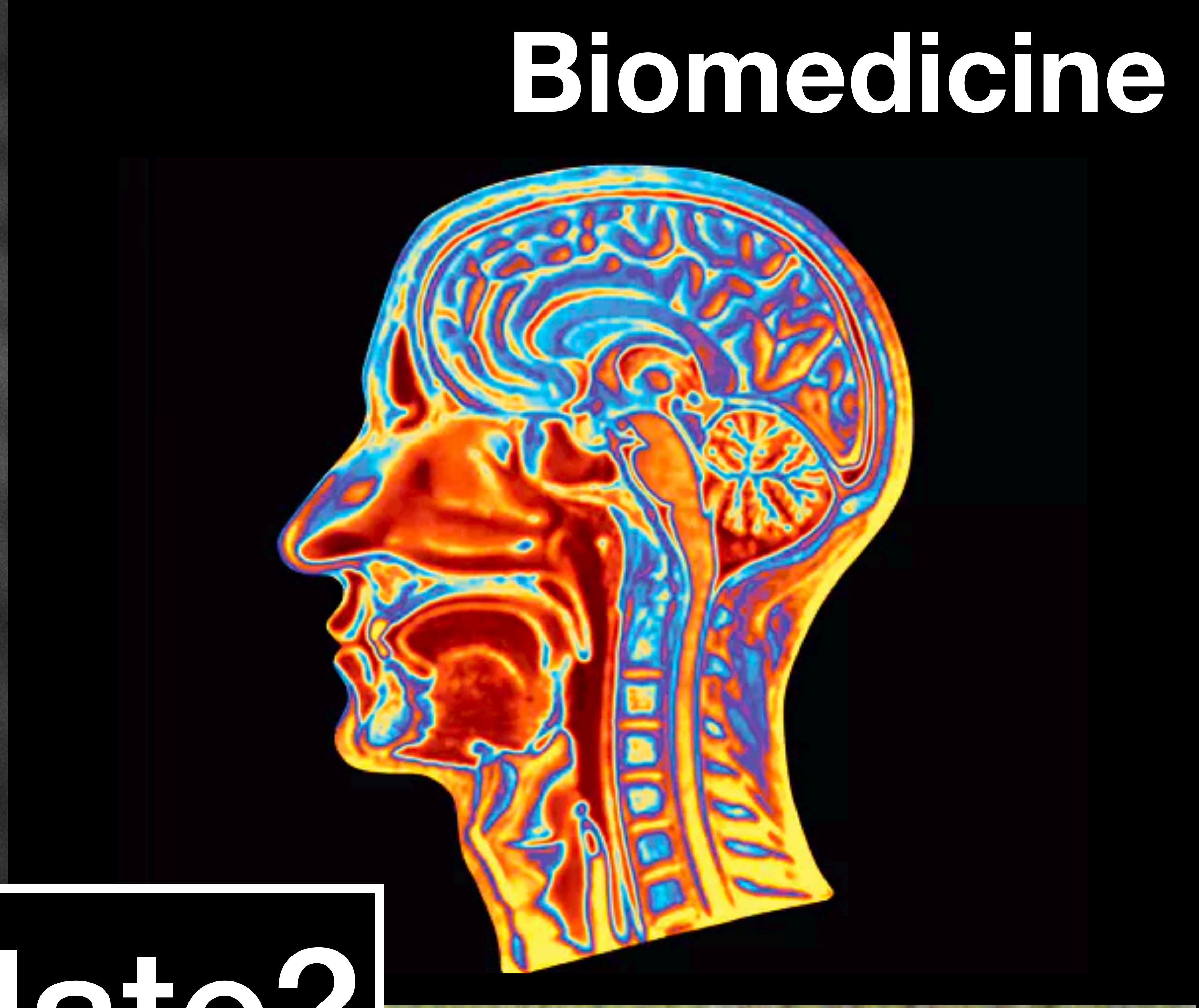
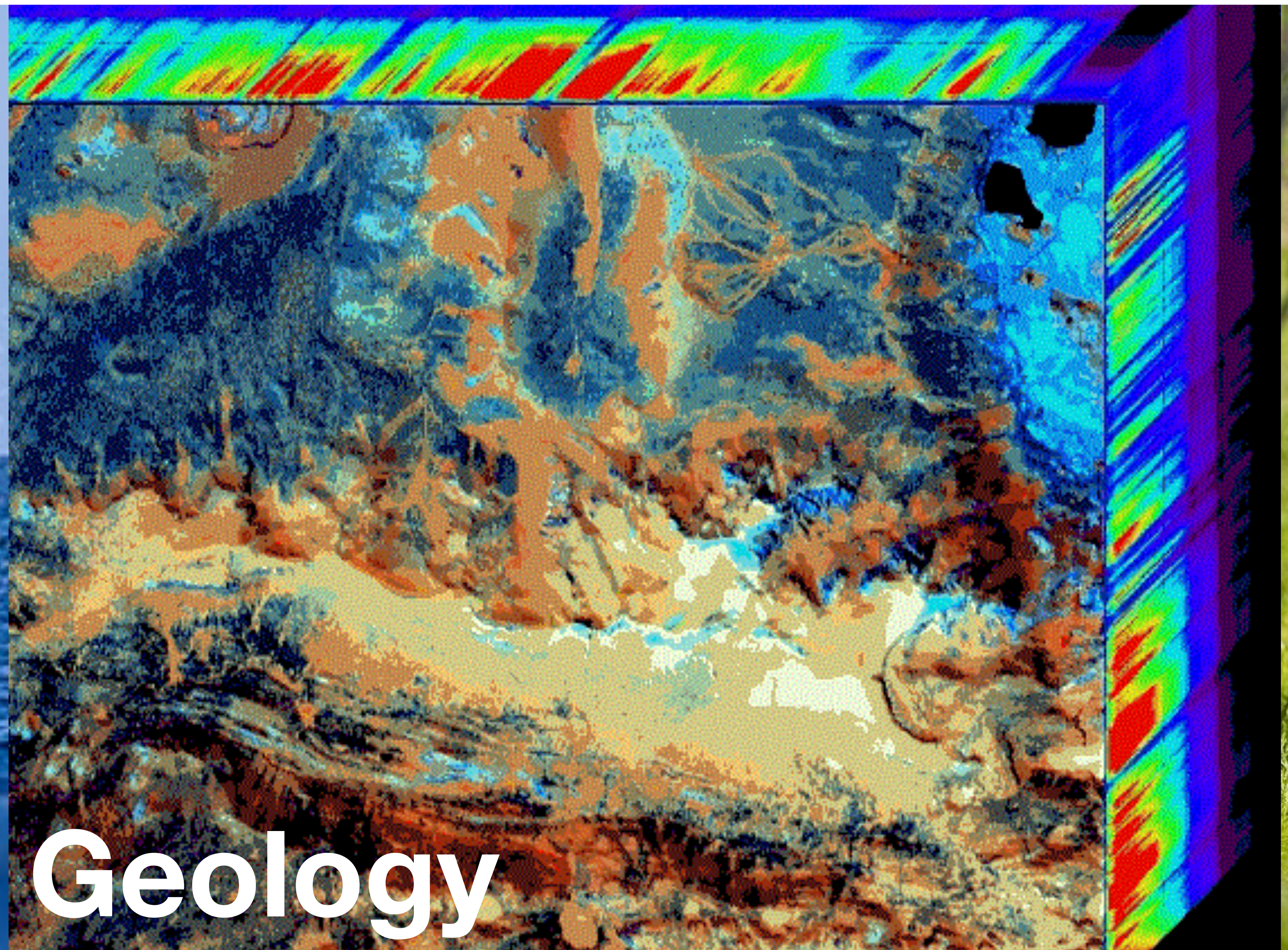
A Spotlight presentation at NeurIPS 2018.



How do things oscillate?



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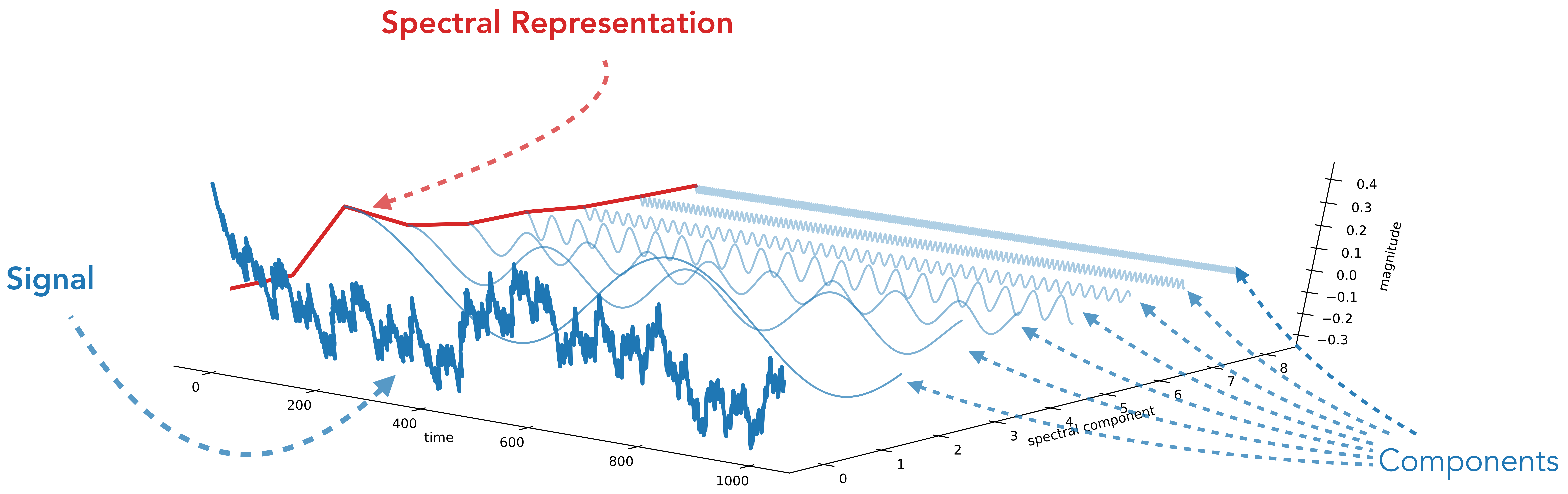
What is the Spectral Representation?

It is to express the distribution of the power of a signal across the frequency spectrum.

Intuitively, this is equivalent to decompose the signal into periodic components



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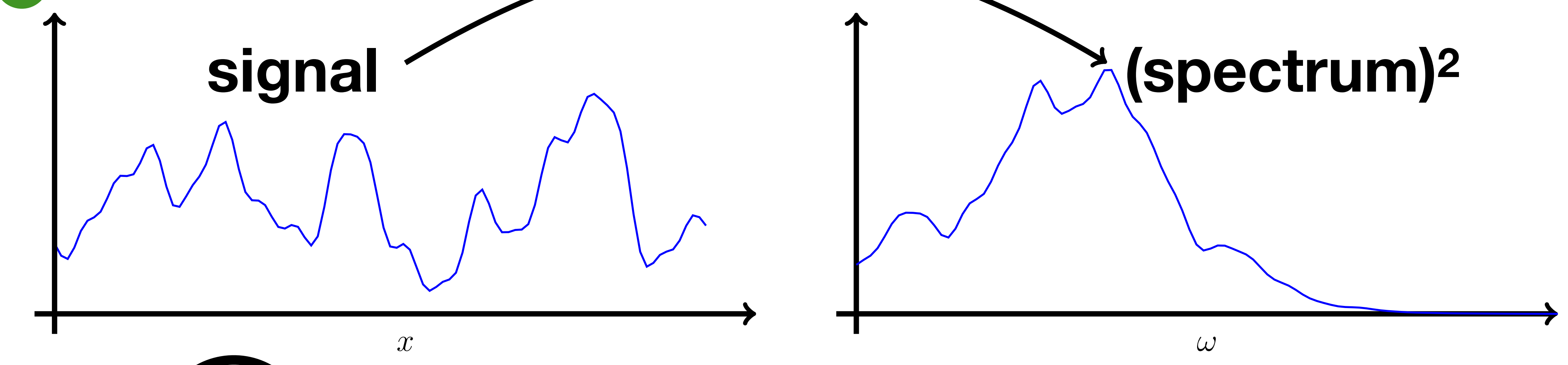
Def: Spectral Estimation

To estimate the spectral representation only using corrupted/partial observations

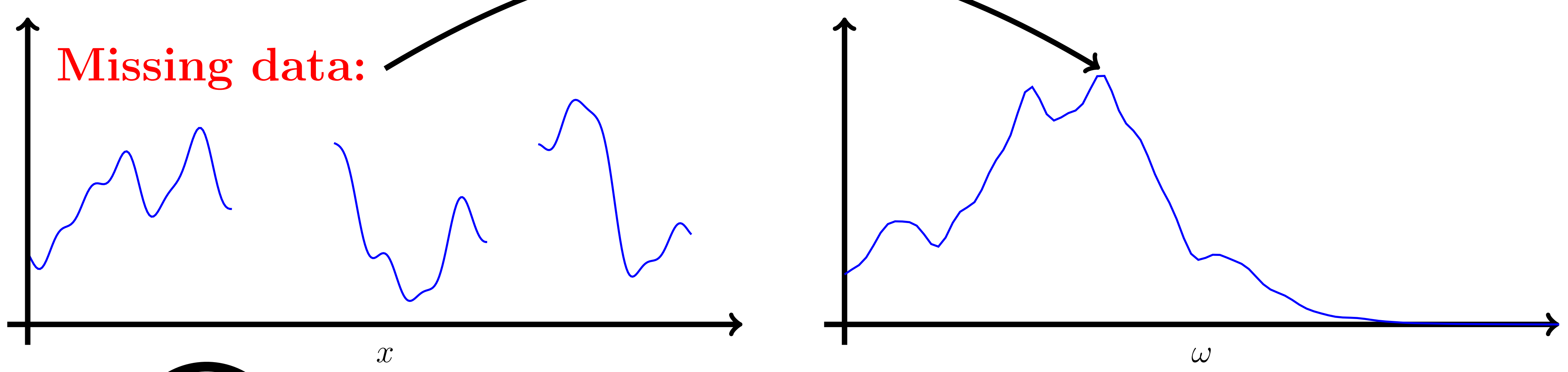
1 Theory

$f(x) \to \mathcal{F}(\omega) = \int f(x)e^{-j2\pi x\omega}dx \to |\mathcal{F}(\omega)|^2$



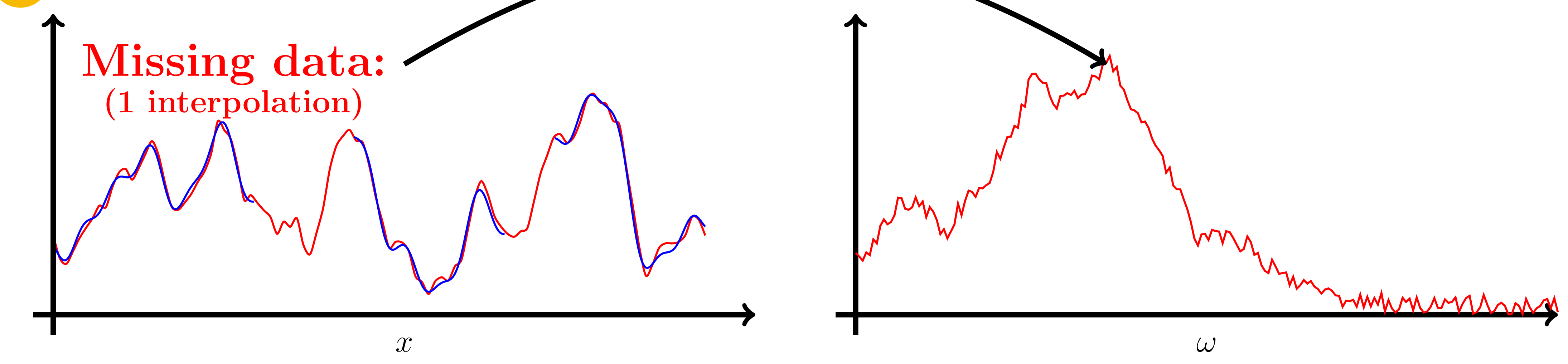
2 Real World

$f(x) \to$ ~~$\mathcal{F}(\omega) = \int f(x)e^{-j2\pi x\omega}dx$~~ $\to |\mathcal{F}(\omega)|^2$



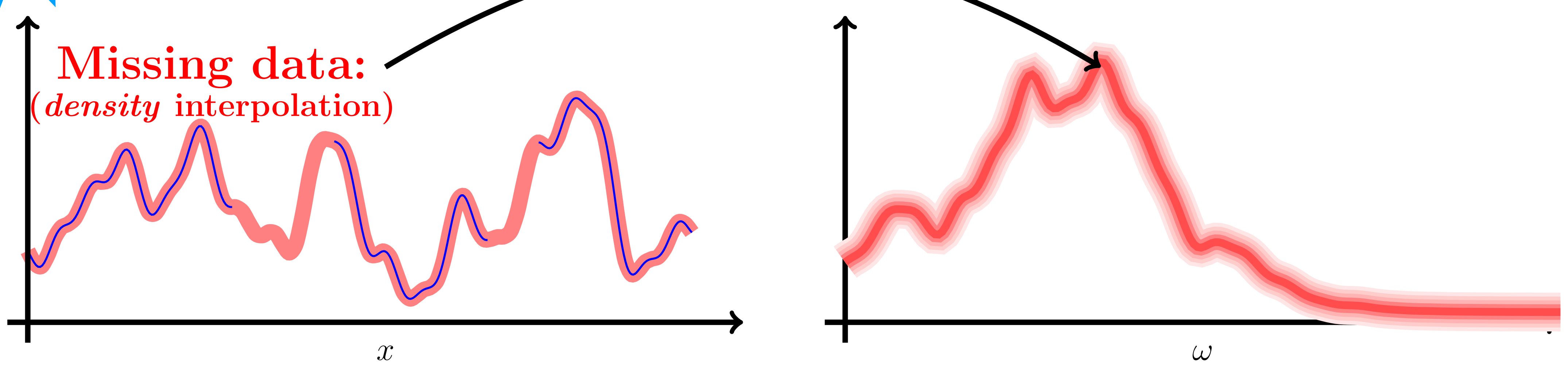
3 Classic

observations $\to \hat{f}(x) \to \hat{\mathcal{F}}(\omega) = \int \hat{f}(x)e^{-j2\pi x\omega}dx \to \left|\hat{\mathcal{F}}(\omega)\right|^2$



4 Ours!

observations $\to p(f(x)|\text{obs}) \to p(\mathcal{F}(\omega)|\text{obs}) \to p\left(|\mathcal{F}(\omega)|^2|\text{obs}\right)$



F. Tobar. Bayesian nonparametric spectral estimation. NeurIPS 2018.

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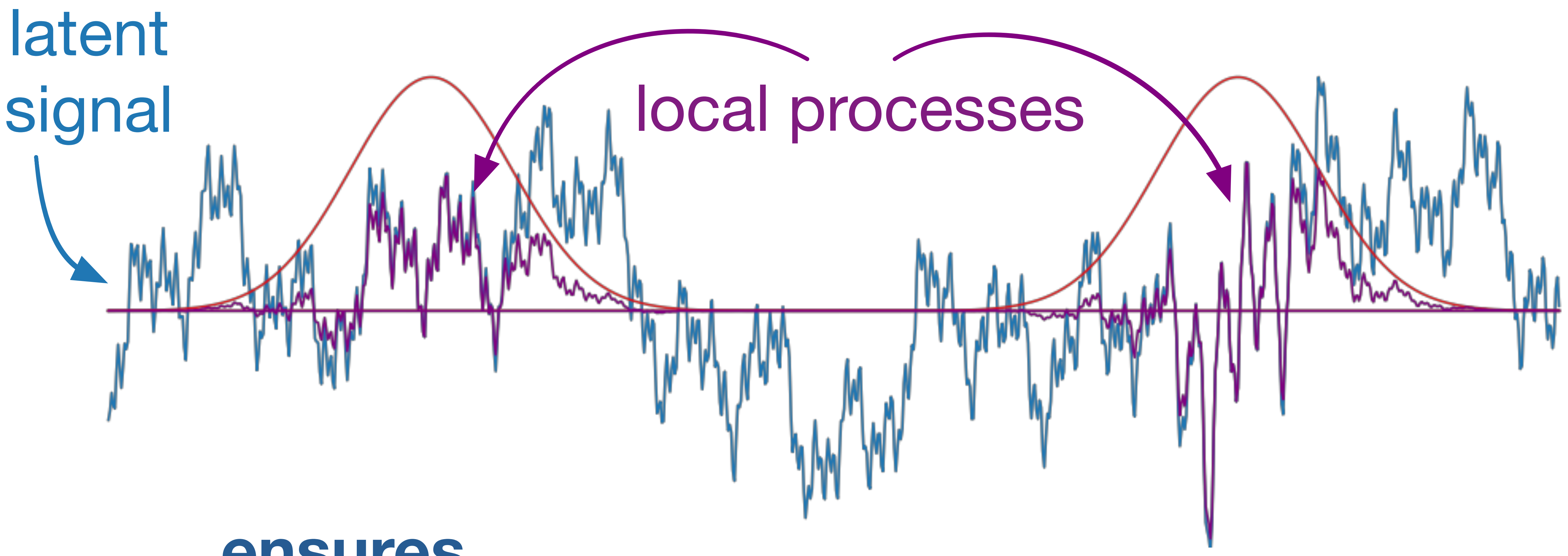
Proposed model:
Bayesian nonparametric spectral estimation

latent signal: $f(t) \sim \mathcal{GP}(0, K)$

observations: $y(t_i) = f(t_i) + \eta_i, \eta_i \sim \mathcal{N}(0, \sigma_n^2), \ \forall i = 1, \dots, N$

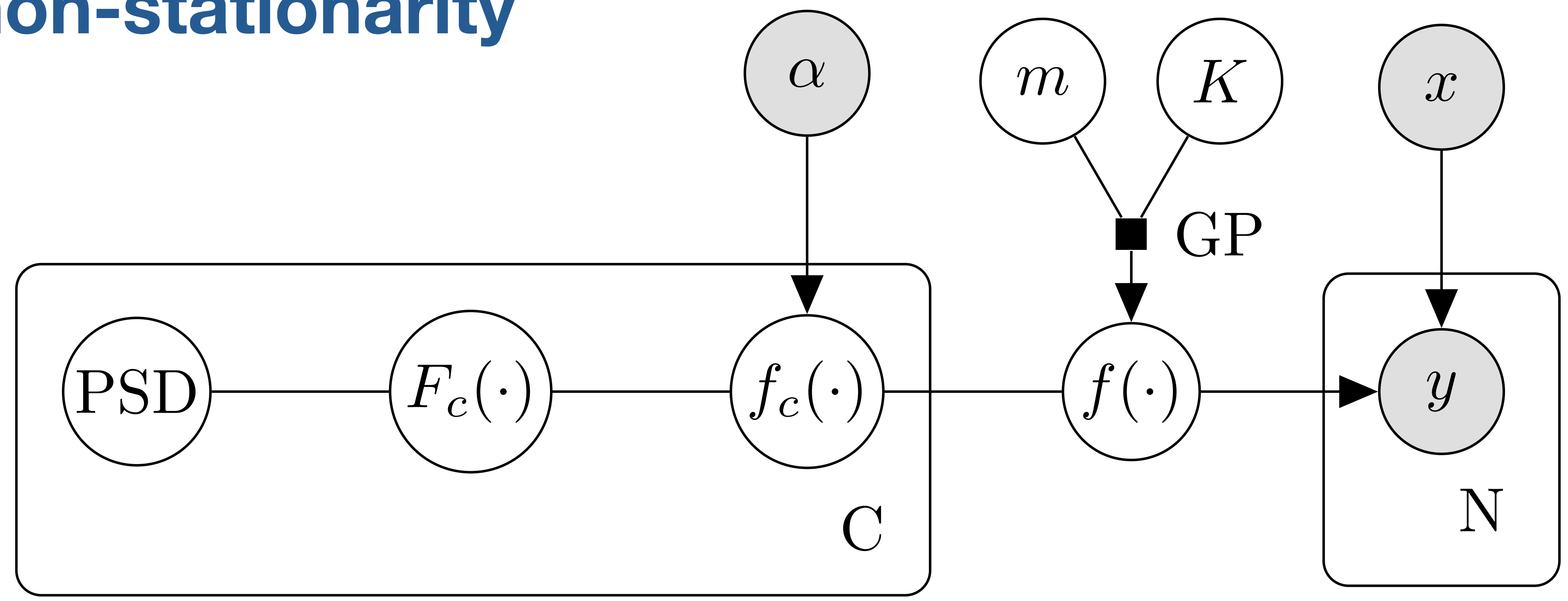
windowed signal: $f_c(t) = e^{-\alpha t^2} f(t - c)$

local spectrum: $F_c(\xi) \triangleq \mathcal{F}\{f_c(t)\} = \mathcal{F}\left\{f(t-c)e^{-\alpha t^2}\right\} = \int_{\mathbb{R}} f(t-c)e^{-\alpha t^2} e^{-j2\pi\xi t}\mathrm{d}t$



handles non-stationarity

ensures Lebesgue measurability



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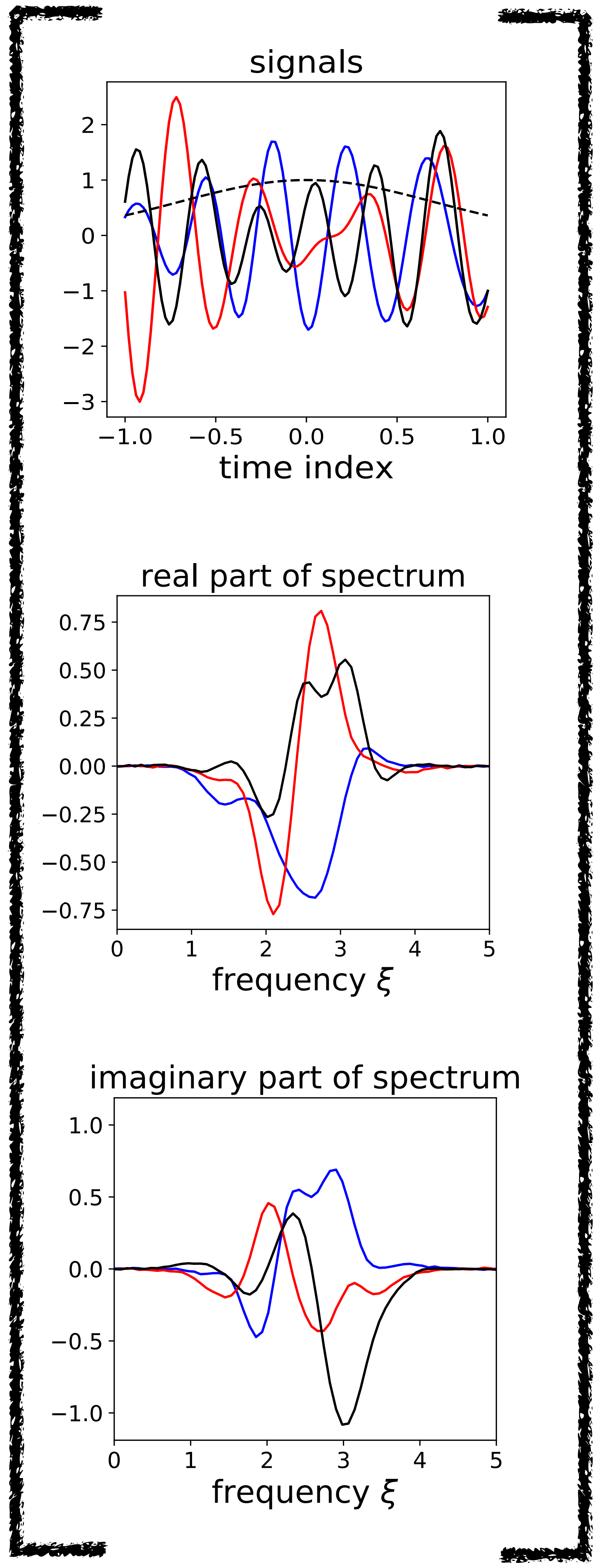
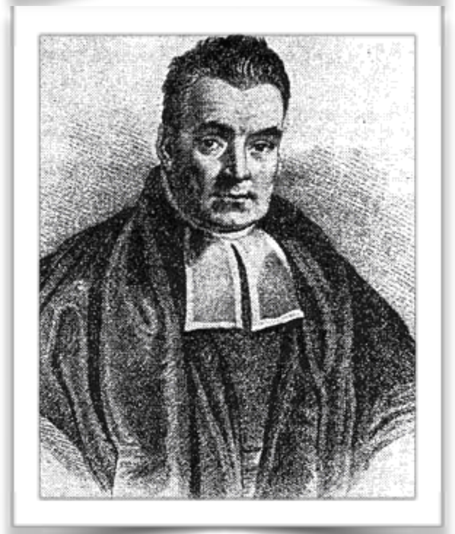
Key findings

Proposition 1 *If a stationary signal $f(t)$ is a Gaussian process, then, $f(t)$ together with its observations y and local spectrum $F_c(\xi)$ are jointly-Gaussian processes with explicit covariance kernels.*





~ MultiOutput GaussianProcess!

Spectral estimation?
Find: $p(F_c|y)$ - (Gaussian)

Reconstruct signal from spectrum?
Find: $p(f|F_c)$ - (Gaussian)

Missing/corrupted observations?
Place errorbars on spectrum

Looking for periodicities?
Optimise $p((F_c)^2|y)$ - (Chi-squared)

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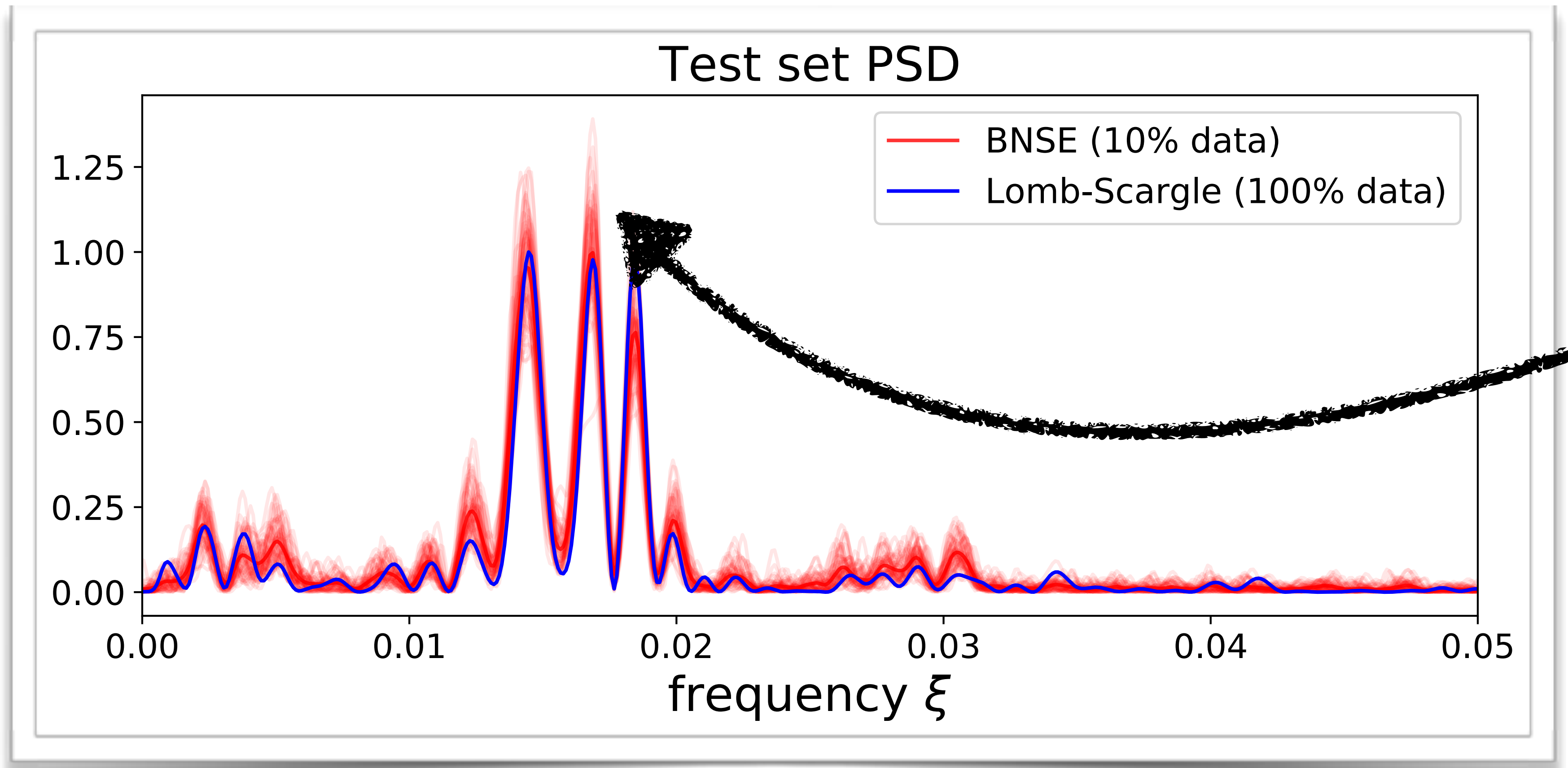
Two Experiments

Spectral uncertainty (a heart-rate signal)

i) only use 10% of signal
ii) find error bars of spectrum **(red)**
iii) compare against ground truth **(blue)**

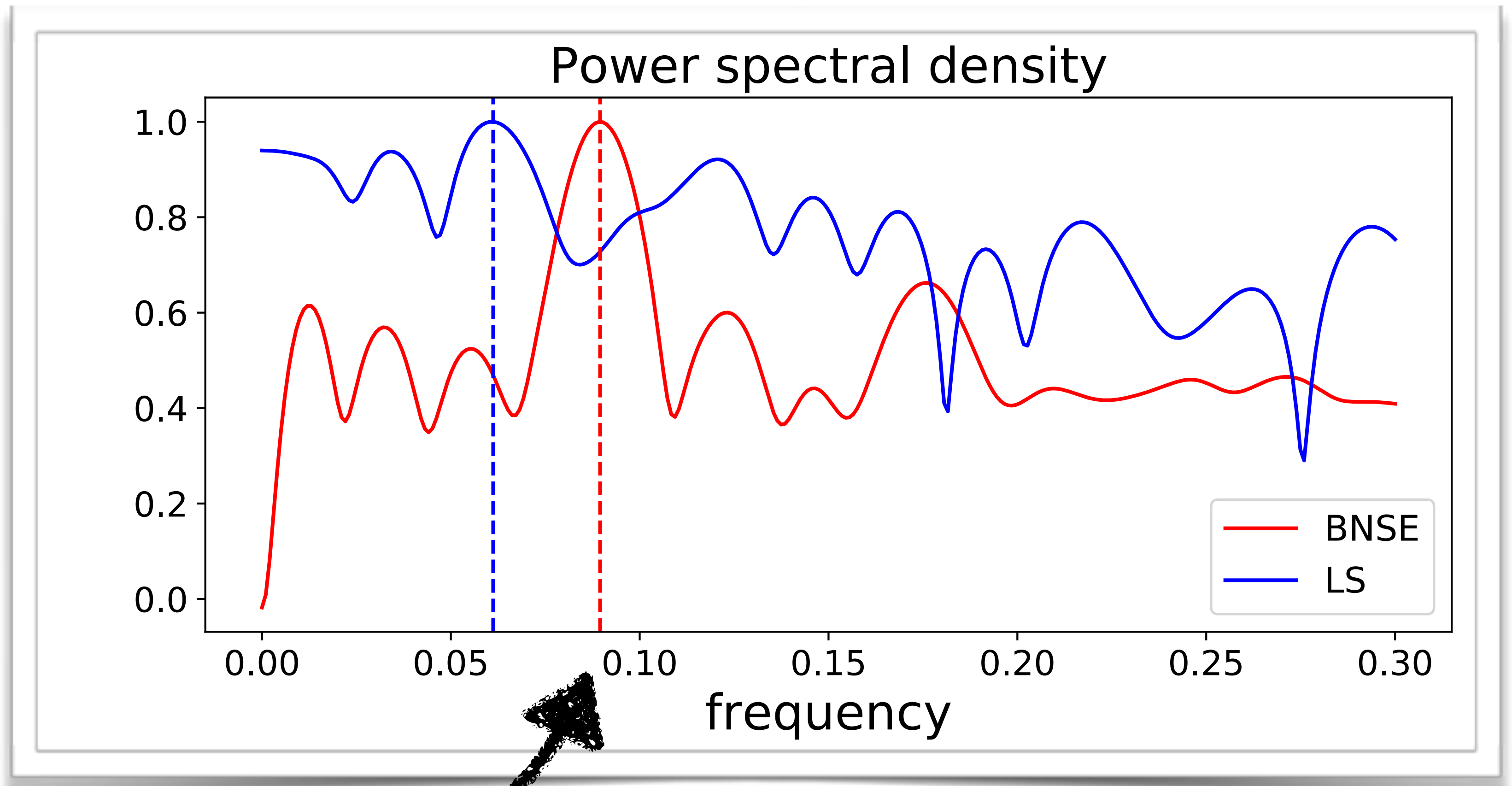
Finding periodicities (an astronomical series)

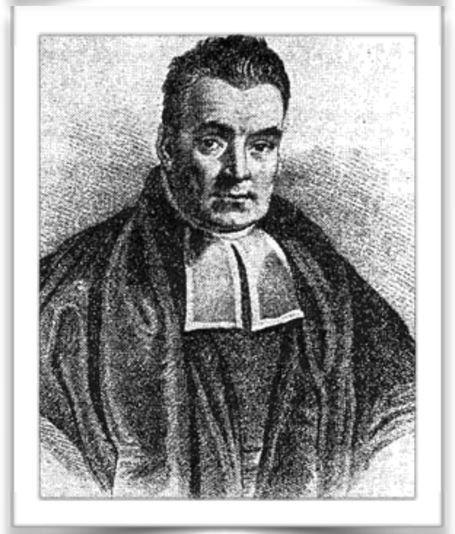
i) implement Lomb-Scargle **(blue)**
ii) compute posterior spectrum **(red)**
iii) just optimise!



Error bars!

True period!





For more

Bayesian nonparametric spectral estimation

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Thank you!