(Probably) Concave Graph Matching

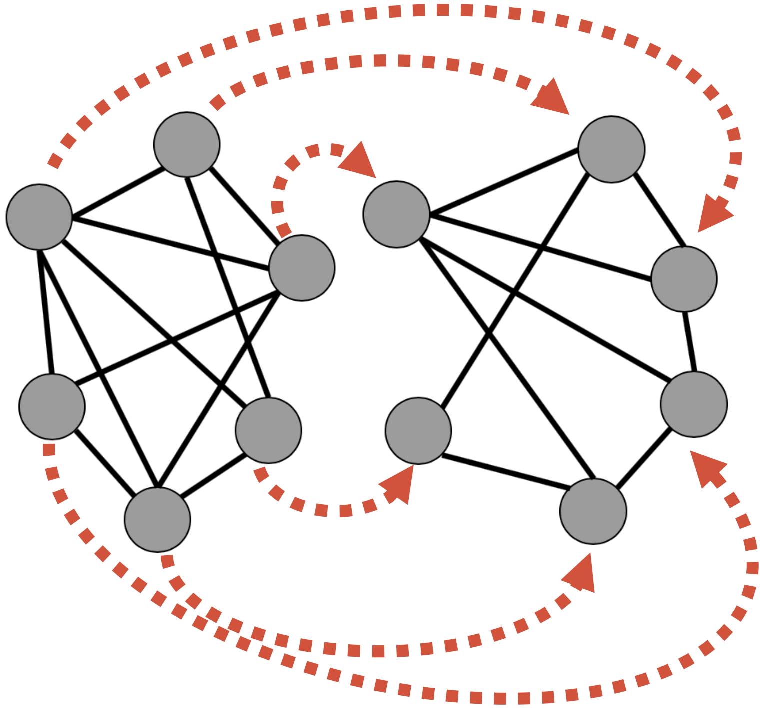
Haggai Maron and Yaron Lipman

Weizmann Institute of Science

מכון ויצמן למדע

WEIZMANN INSTITUTE OF SCIENCE

Graph Matching



$$\min_{X \in \Pi_n} -\mathrm{tr}(\boldsymbol{A} X \boldsymbol{B} X^T)$$

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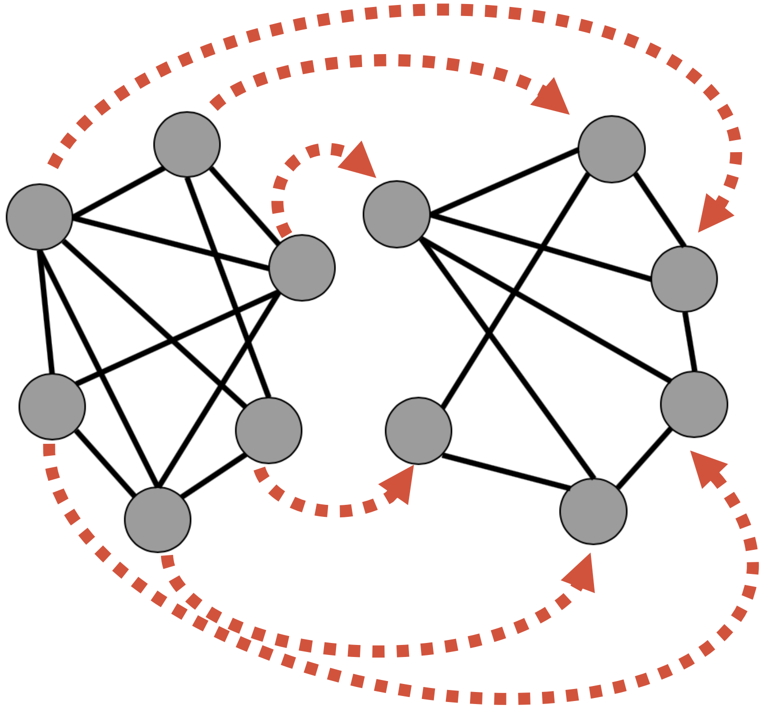

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DS

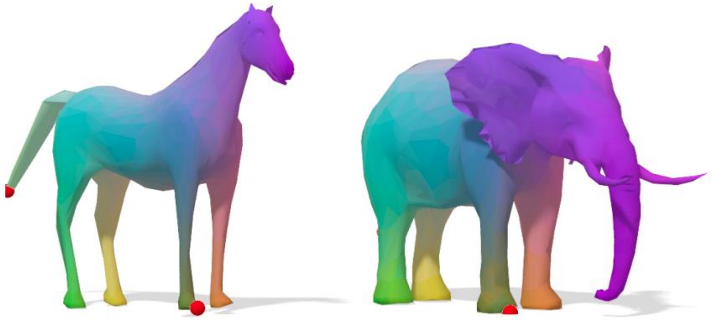
Previous Work

- Superiority of the indefinite relaxation
  - [Lyzinski et al. PAMI 2016]


Graph Matching: Relax at Your Own Risk

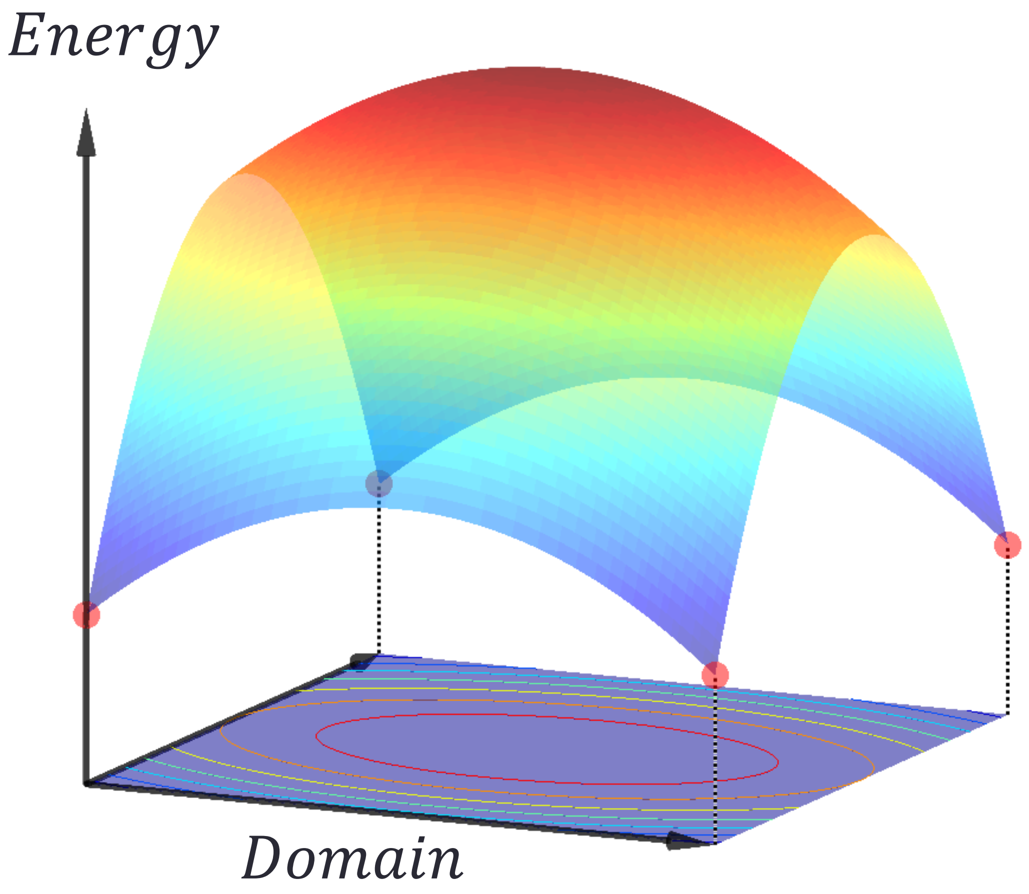
Vince Lyzinski, Donniell E. Fishkind, Marcelo Fiori, Joshua T. Vogelstein, Carey E. Priebe, and Guillermo Sapiro, *Fellow, IEEE*

- Efficient graph matching via concave energies
  - [Vestner et al. CVPR 2017, Boyarski et al. 3DV 2017]

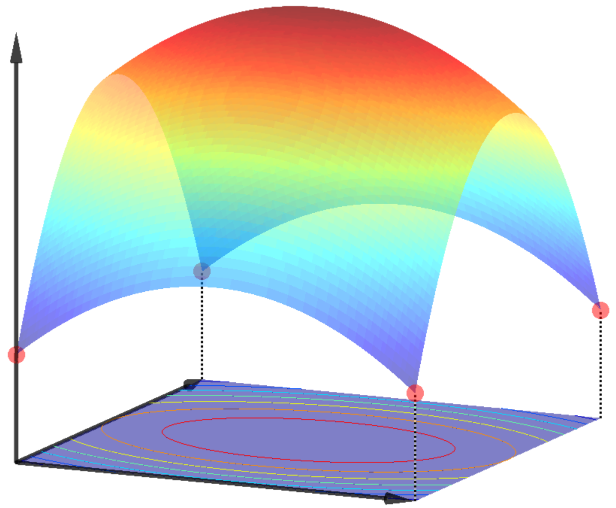


Advantages of Concave Relaxations

- All local minima are permutation matrices



Many important graph matching problems are concave!



Which A, B give rise to concave relaxations?

Concavity of Indefinite Relaxation

- **Theorem:** It is sufficient that

$$A = \Phi\left(x_i - x_j\right), B = \Psi(y_i - y_j)$$

where Φ, Ψ are positive definite functions of order one.

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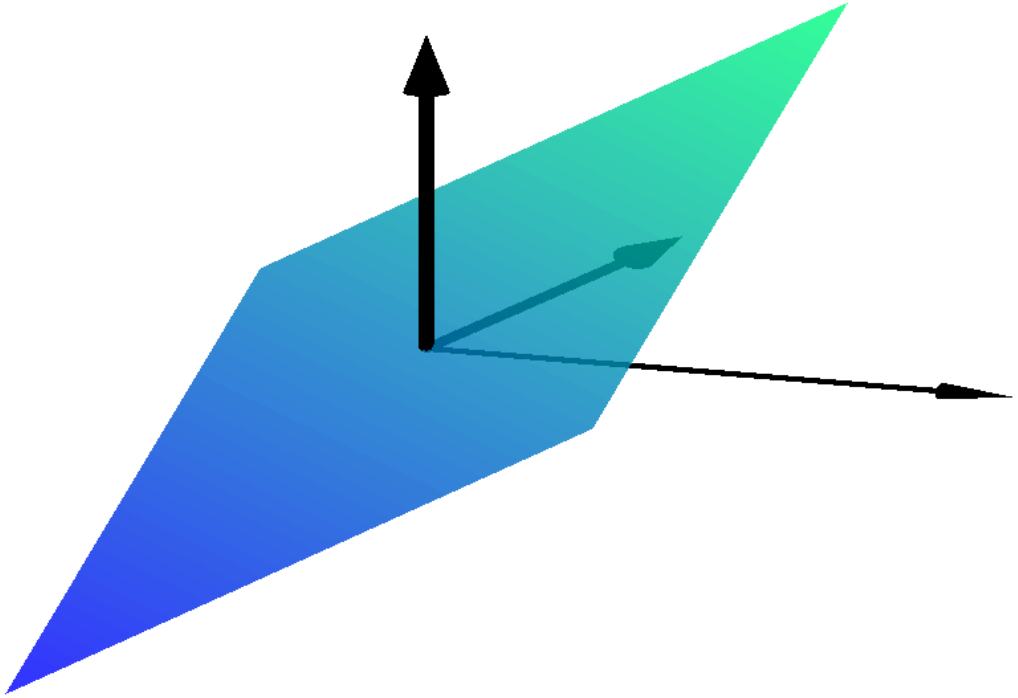

Concave Energies

Euclidean distance in any dimension

- Mahalanobis distances
- Spectral graph distances
- Matching objects with deep descriptors

$$A_{ij} = ||x_i - x_j||_2$$

Spherical distance in any dimension
[bogomolny, 2007]

$$A_{ij} = d_{S^n}\,(x_i, x_j)$$

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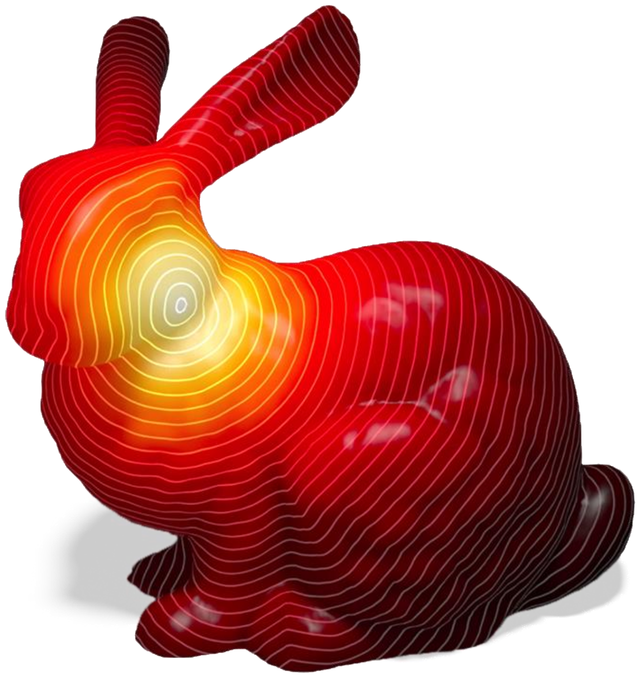

Image taken from Crane et al. 2017

Probably Concave Energies

- **Theorem (upper bound on the probability of convex restriction)**

Let $M \in \mathbb{R}^{m \times m}$ and $D \leq \mathbb{R}^m$ a uniformly sampled d-dimensional subspace, then:

$$Pr\left[M\Big|_D \succ 0\right] \leq \min_t \prod_{i=1}^{n} (1 - 2t\lambda_i)^{-d/2}$$

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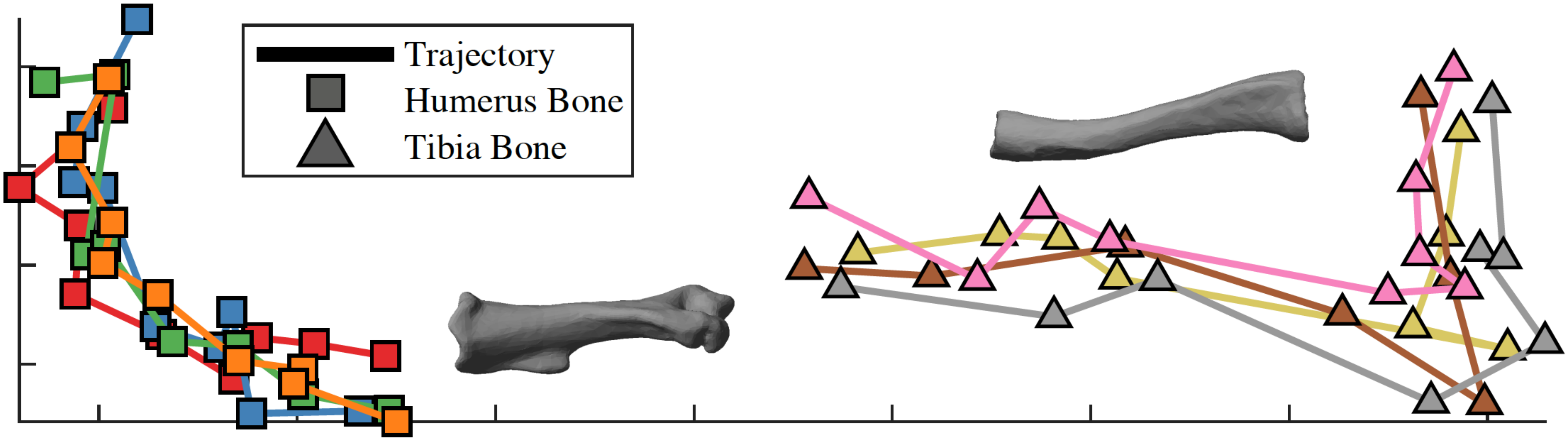
$$\mathrm{diag}(\overbrace{-1, -1, \ldots, -1}^{0.51m}, \overbrace{1, 1, \ldots, 1}^{0.49m})$$

Applications



Conclusion

- A large family of concave or probably concave relaxations
- Checking probable concavity with eigenvalue bound
- Extension of [Lyzinsky et al. 2016] to practical matching problems

The End

- Support
  - ERC Grant (LiftMatch)
  - Israel Science Foundation

- Thanks for listening!