

An Asymptotically Optimal Batched Algorithm for the Dueling Bandit Problem

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Motivation: Web-search Ranking

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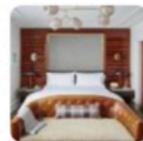
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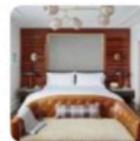
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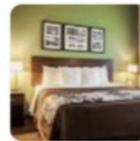
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Can extract pairwise comparisons
(Radlinski et al., 2008)

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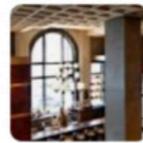


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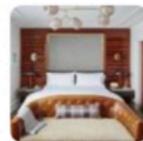
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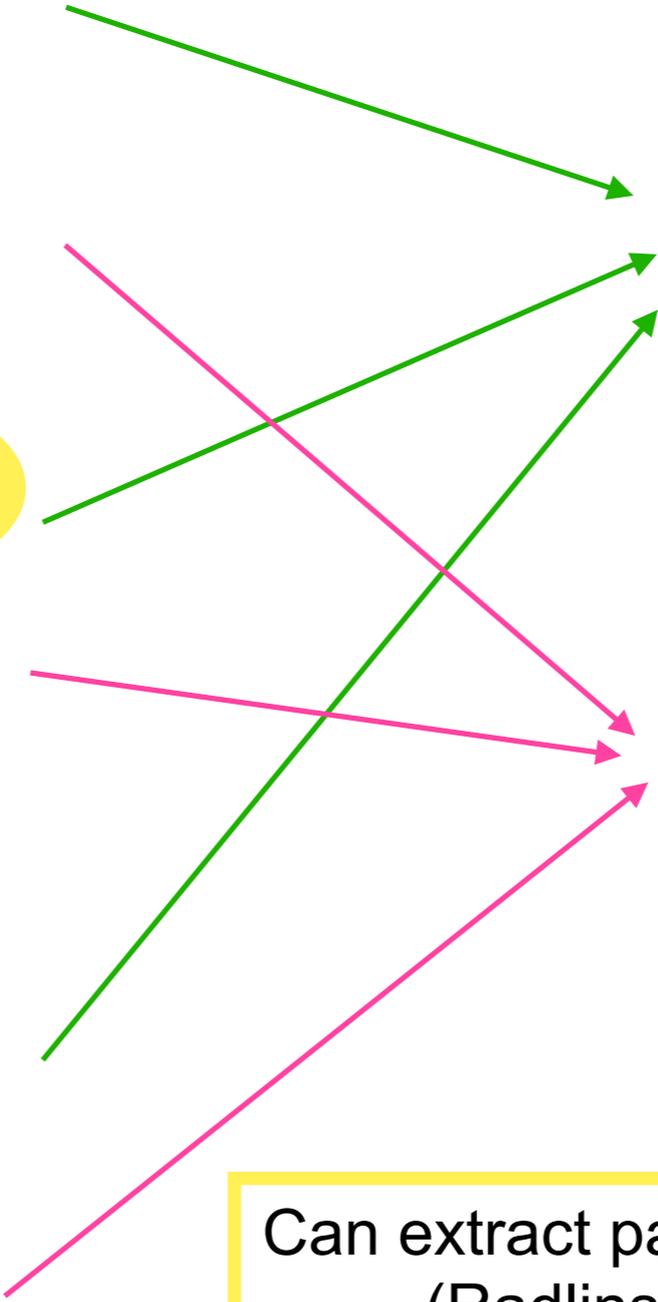
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Click!

Ranking A

Ranking B

Can extract pairwise comparisons (Radlinski et al., 2008)



Dueling Bandits

- K arms, time horizon T
- In trial $t \in T$:
 - select pair (i_t, j_t)
- Observe noisy comparison:
 - $P(i \text{ beats } j) = p_{i,j}$; comparisons are **independent**
 - $p_{i,j} = \frac{1}{2} + \Delta_{i,j}$
- **Condorcet assumption**: $\exists i^* : p_{i^*,i} \geq 1/2 \forall i$
- Regret at time t : $r(t) = \Delta_{i^*,i_t} + \Delta_{i^*,j_t}$
- Total regret: $R(T) = \sum_t r(t)$

Goal: perform noisy comparisons that have low regret

Batched Dueling Bandits

- Instance of Dueling Bandits
- Number of batches, B

Goal: perform B batches of noisy comparisons that have low regret

Notation: $\Delta_j = \Delta_{i^*,j}$ $\Delta_{\min} = \min_{j:\Delta_j \neq 0} \Delta_j$

Theorem 1: Given $B \geq 1$, we give an algorithm that uses at most B rounds and has expected regret (under Condorcet assumption):

$$\mathbb{E}[R(T)] = O\left(T^{1/B} \cdot \frac{K^2 \log(K)}{\Delta_{\min}^2} \cdot \log\left(\frac{\log(K)}{\Delta_{\min}}\right)\right) + O\left(T^{2/B} \cdot K^2\right) + \sum_{j \neq i^*} O\left(\frac{T^{1/B} \log(KT)}{\Delta_j}\right)$$

Batched Dueling Bandits

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• $O(K^2 \log^2(K)) + O(K \log(T))$ asymptotic regret in $O(\log(T))$ rounds!

- best sequential bound: $O\left(\frac{K^2}{\Delta_{\min}^2}\right) + \sum_{j: \Delta_j > 0} O\left(\frac{\log(T)}{\Delta_j}\right)$

- our result (nearly) matches best sequential result using only $O(\log(T))$ rounds

Techniques

- 1) i^* can be **trapped** using few adaptive rounds (in expectation)
- 2) Once trapped, use i^* to **eliminate sub-optimal arms**

Conclusion

- Studied the **batched dueling bandits** problem under the Condorcet assumption
- Give an algorithm that achieves a **trade-off b/w #batches and regret**
 - $O(K^2 \log^2(K)) + O(K \log(T))$ asymptotic regret in $O(\log(T))$ rounds!
- Algorithms relies on:
 - 1) trapping best arm
 - 2) Using trapped arm to eliminate sub-optimal arms

THANK YOU!