



compute. collaborate. create.

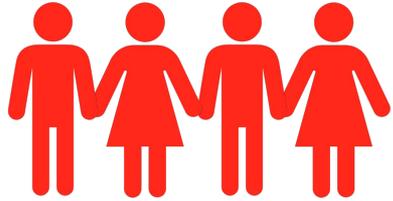
Falsification before Extrapolation in Causal Effect Estimation

Zeshan Hussain*, Michael Oberst*, Ming-Chieh Shih*, David Sontag

*equal contribution, by alphabetical order

Motivating Example

Randomized Controlled Trial (RCT)



Motivating Example

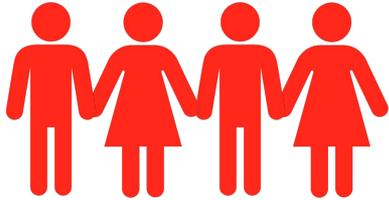
Randomized Controlled Trial (RCT)



RCTs often fail to include all types of patients (e.g. )

Motivating Example

Randomized Controlled Trial (RCT)

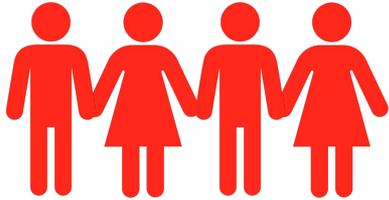


Real-world example: pregnant women were not included in initial COVID-19 trials¹

1] Dagan, Noa, et al. "Effectiveness of the BNT162b2 mRNA COVID-19 vaccine in pregnancy." *Nature medicine* 27.10 (2021): 1693-1695.

Motivating Example

Randomized Controlled Trial (**RCT**)



Observational Study (**OS**)



Observational studies contain a more **diverse cohort**, but may suffer from e.g. **unobserved confounding**.



Motivating Example

RCT



Motivating Example

Observational
Study #1

Observational
Study #2

RCT



Motivating Example

Observational
Study #1

Observational
Study #2

RCT

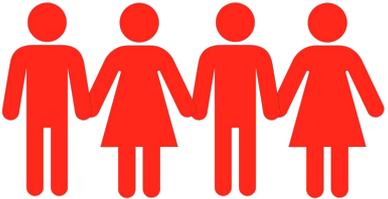


Motivating Example

Observational Study #1

Observational Study #2

RCT

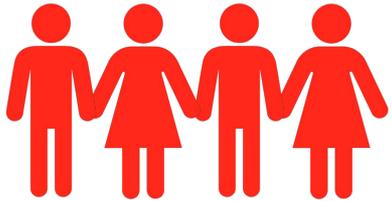


Motivating Example

Observational
Study #1

Observational
Study #2

RCT

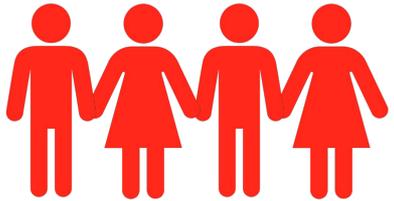
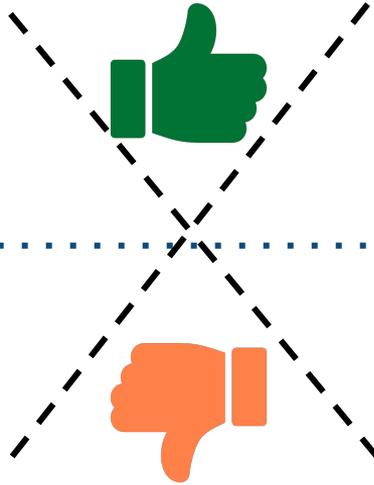


Motivating Example

Observational
Study #1

Observational
Study #2

RCT



Motivating Example

Observational
Study #1

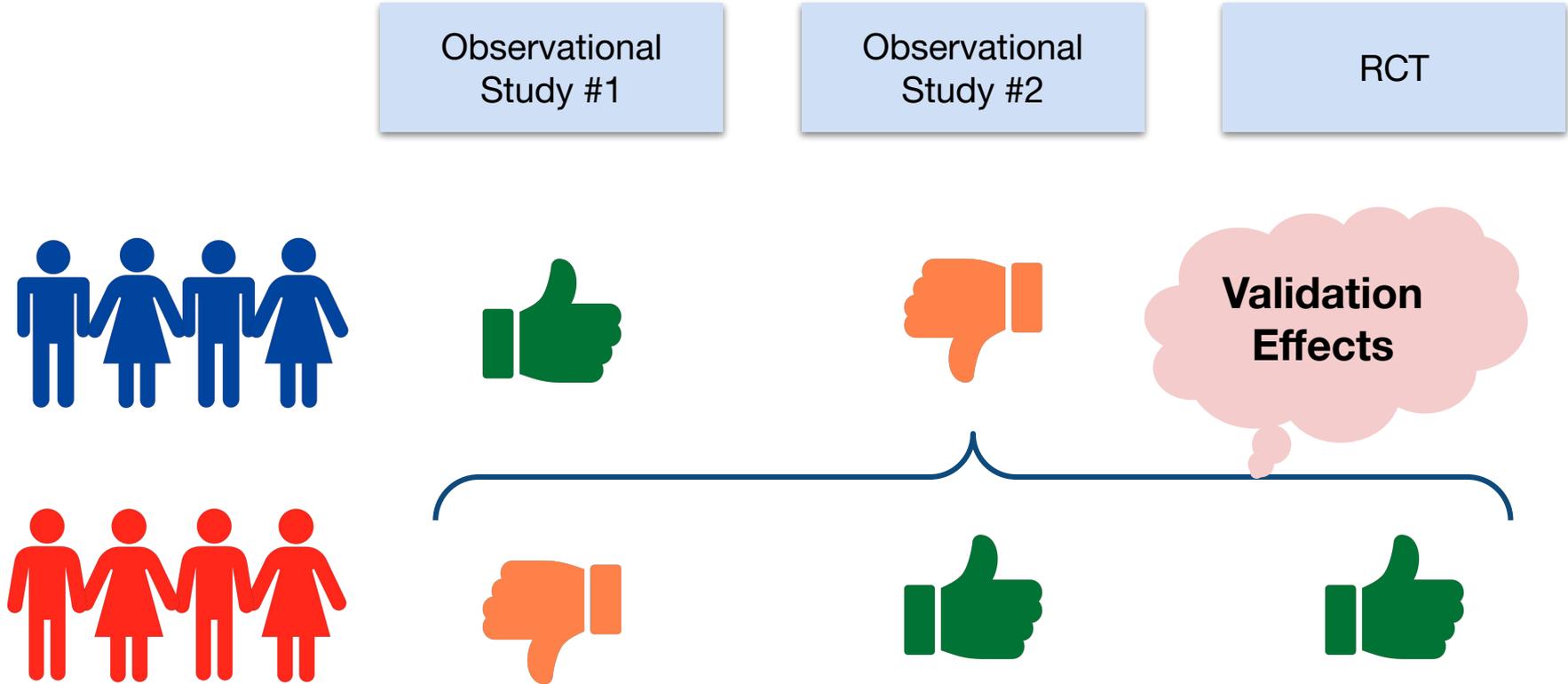
Observational
Study #2

RCT

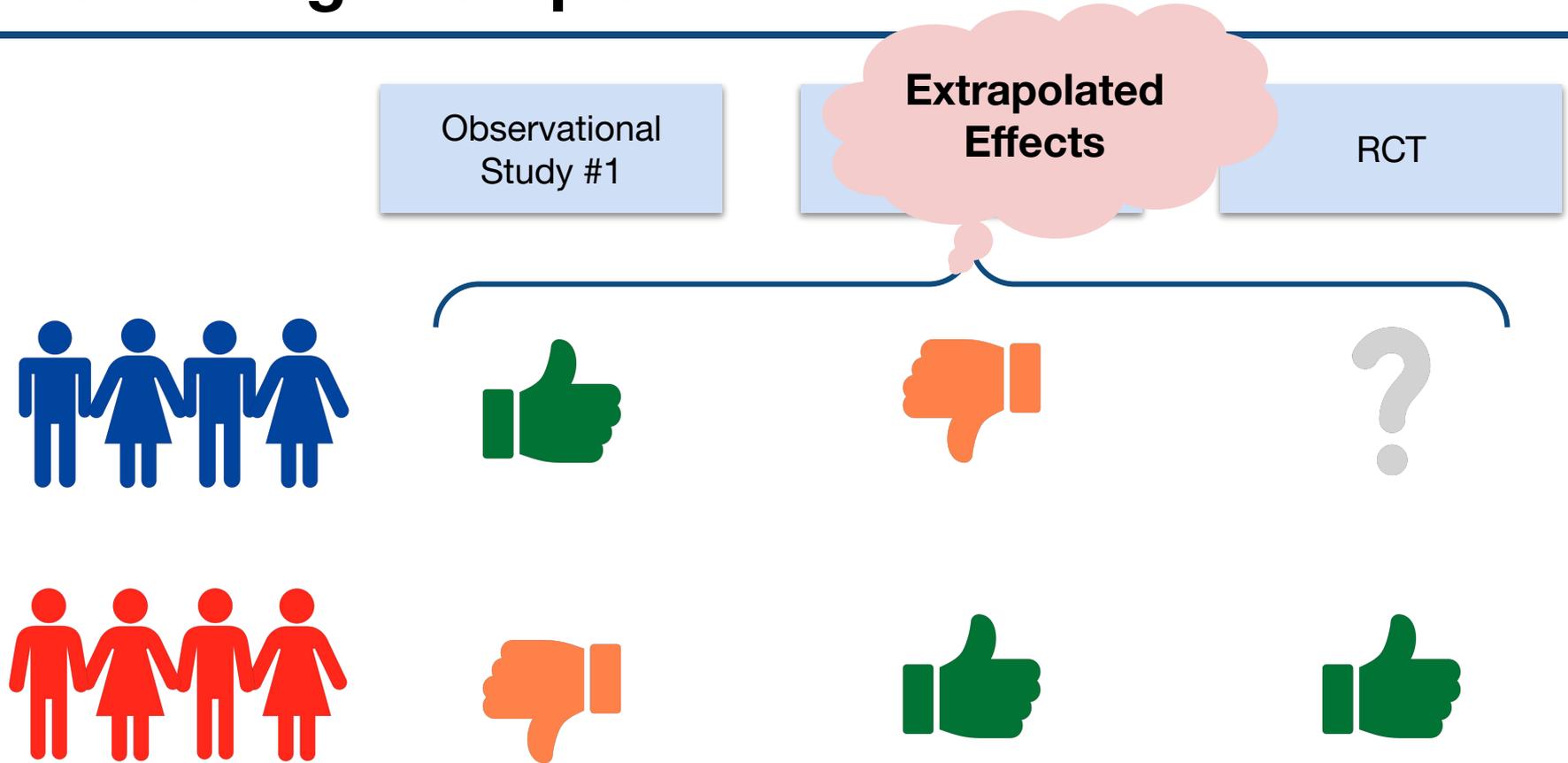


Main idea: Reject observational studies
that fail to replicate RCT results

Motivating Example



Motivating Example



Contributions

Our
Approach

1

Falsification of
observational estimates

Contributions

Our
Approach

1

Falsification of
observational estimates

Use framework of **hypothesis
testing**

Contributions

Our
Approach

1

Falsification of
observational estimates

Use framework of **hypothesis
testing**

Reject estimators that do not
replicate **RCT estimates**

Contributions

Our
Approach

1

Falsification of
observational estimates

2

Pessimistic Combination
of Confidence Intervals

Take the **union** over all the
intervals of the **accepted
estimators**.

Formalizing Falsification

Observational Study #1

Observational Study #2

RCT

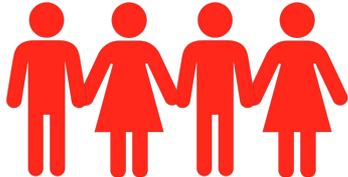


Formalizing Falsification

Observational Study #1

Observational Study #2

RCT



We refer to the treatment effect in each group i as the **group average treatment effect (GATE): τ_i** .

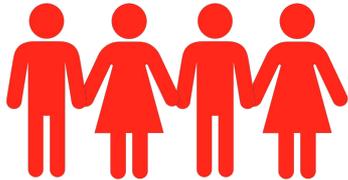


Formalizing Falsification

Observational Study #1

Observational Study #2

RCT



Assumption 1: All observational studies have **support** in all subgroups.

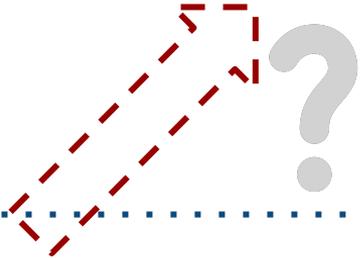


Formalizing Falsification

Observational Study #1

Observational Study #2

RCT



Assumption 2: RCT is a consistent estimator for each
GATE: $\hat{\tau}_i(0) \xrightarrow{p} \tau_i$



Formalizing Falsification

Observational Study #1

Observational Study #2

RCT



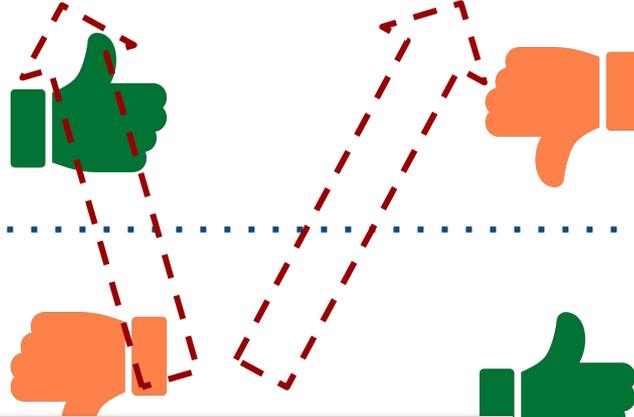
Assumption 3: At least one observational estimator is “correct”, i.e. is **consistent estimator for all GATEs.**

Formalizing Falsification

Observational
Study #1

Observational
Study #2

RCT



Assumption 3: At least one observational estimator is “correct”, i.e. is **consistent estimator for all**

GATEs: $\hat{\tau}_i(k) \xrightarrow{p} \tau_i, k = 1, 2$



Formalizing Falsification

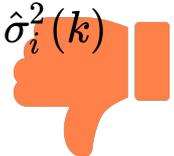
Observational Study #1

Observational Study #2



Will be necessary for hypothesis testing.

Require that estimator for each GATE is **asymptotically normal**



Formalizing Falsification

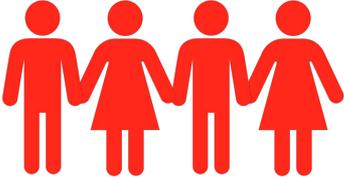
Observational Study #1

Observational Study #2

*Will be necessary for **hypothesis testing**, and we give examples where this is reasonable.*



Require that estimator for each GATE is **asymptotically normal**

$$\sqrt{N_k}(\hat{\tau}_i(k) - \tau_i(k)) / \hat{\sigma}_i(k) \xrightarrow{d} \mathcal{N}(0, 1)$$


Sample size of observational study ($k=1,2$) or RCT ($k=0$)

$\hat{\sigma}_i^2(k)$ is estimate of variance, converges in probability to asymptotic variance



Formalizing Falsification

Observational Study #1

Observational Study #2

*We demonstrate asymptotic normality of GATE estimators with **transportation**.*



Require that estimator for each GATE is **asymptotically normal**



Hypothesis Test Construction



Observational
Study #1



Observational
Study #2



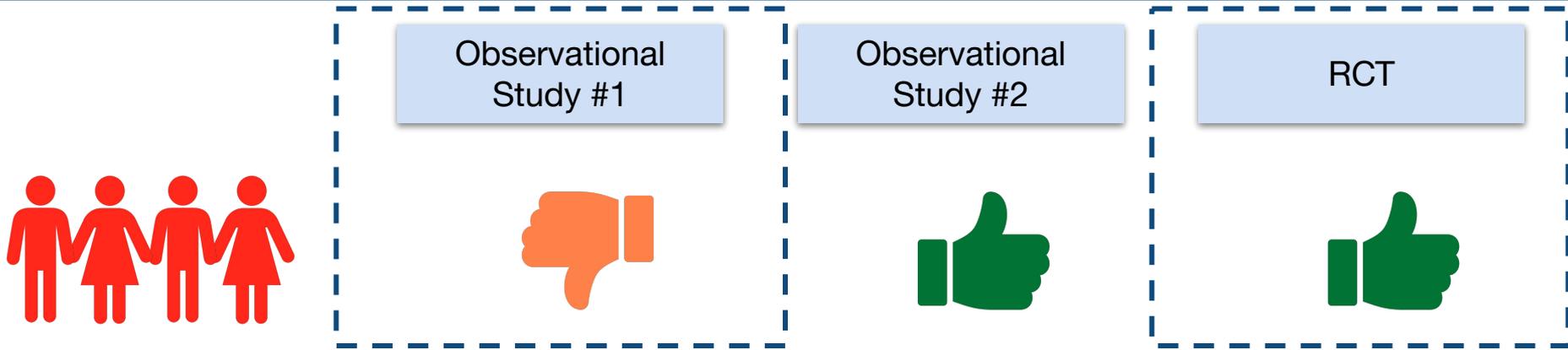
RCT



$$H_0 : \tau_{\text{red}}(1) = \tau_{\text{red}}$$

Want to perform above
hypothesis test with **asymptotic**
level, α

Hypothesis Test Construction



$$H_0 : \tau_{\text{red}}(1) = \tau_{\text{red}}$$

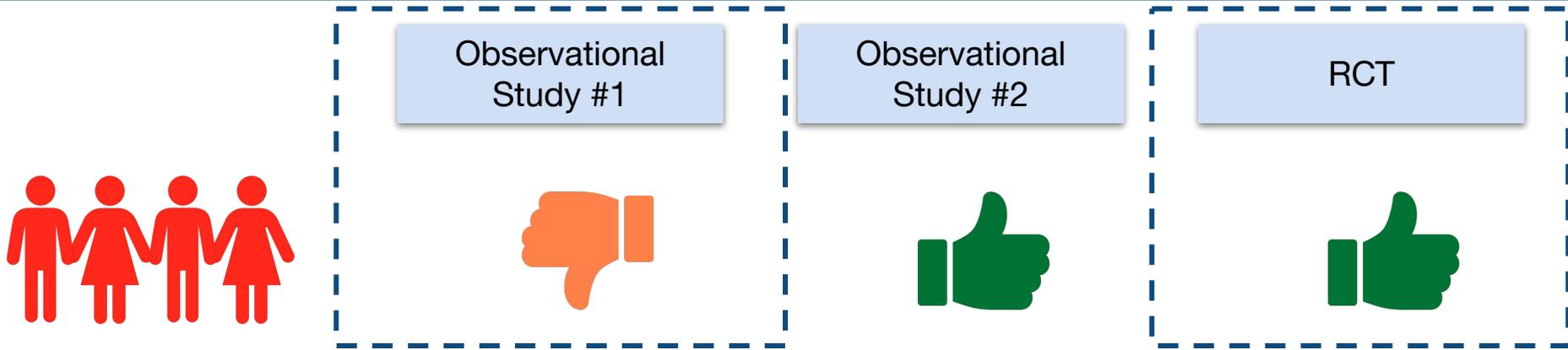
Set equal to 0

We can use the following test statistic, which we show **converges in distribution** to a standard **normal distribution**

$$\hat{T}_N(k = 1, i = \text{red people}) := \frac{(\hat{\tau}_i(1) - \hat{\tau}_i(0)) - (\tau_i(1) - \tau_i)}{\frac{\hat{\sigma}_i^2(1)}{N_1} + \frac{\hat{\sigma}_i^2(0)}{N_0}}$$

Estimated variance

Hypothesis Test Construction



$$H_0 : \tau_{\text{red}}(1) = \tau_{\text{red}}$$

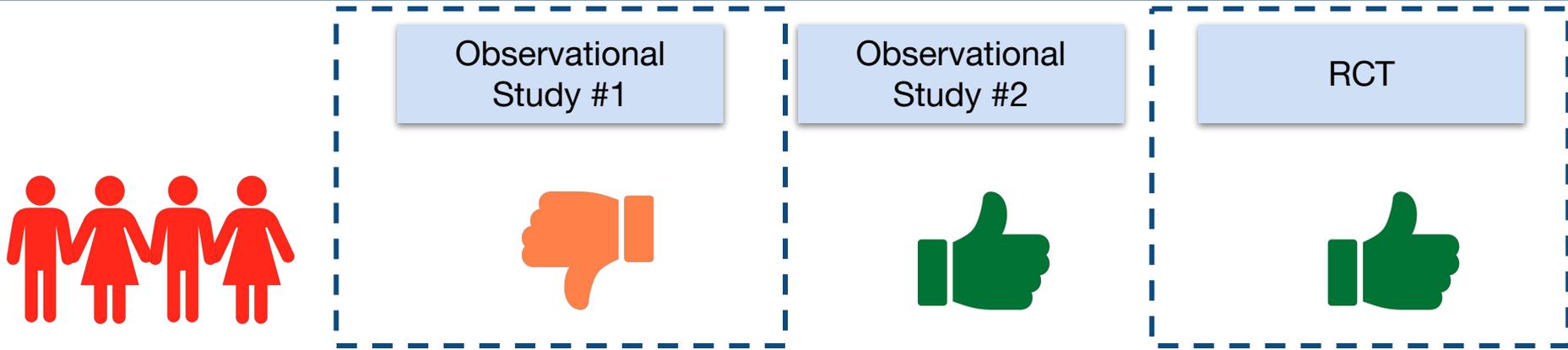
Set equal to 0

Reject the observational study if
 $|\hat{T}_N(k = 1, i = \text{red people})| > z_{\alpha/2}$

$$\hat{T}_N(k = 1, i = \text{red people}) := \frac{(\hat{\tau}_i(1) - \hat{\tau}_i(0)) - (\tau_i(1) - \tau_i)}{\frac{\hat{\sigma}_i^2(1)}{N_1} + \frac{\hat{\sigma}_i^2(0)}{N_0}}$$

Estimated variance

Hypothesis Test Construction



$$H_0 : \tau_{\text{red}}(1) = \tau_{\text{red}}$$

Set equal to 0

Note that we use **Bonferroni correction to control FPR of test**, since we test many subgroups (e.g. red people, blue people, etc.)

$$\hat{T}_N(k = 1, i = \text{red people}) := \frac{(\hat{\tau}_i(1) - \hat{\tau}_i(0)) - (\tau_i(1) - \tau_i)}{\frac{\hat{\sigma}_i^2(1)}{N_1} + \frac{\hat{\sigma}_i^2(0)}{N_0}}$$

Estimated variance

Our Approach

1

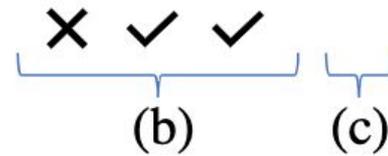
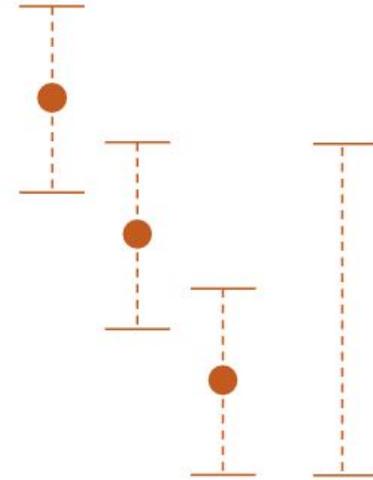
Falsification of
observational estimates

2

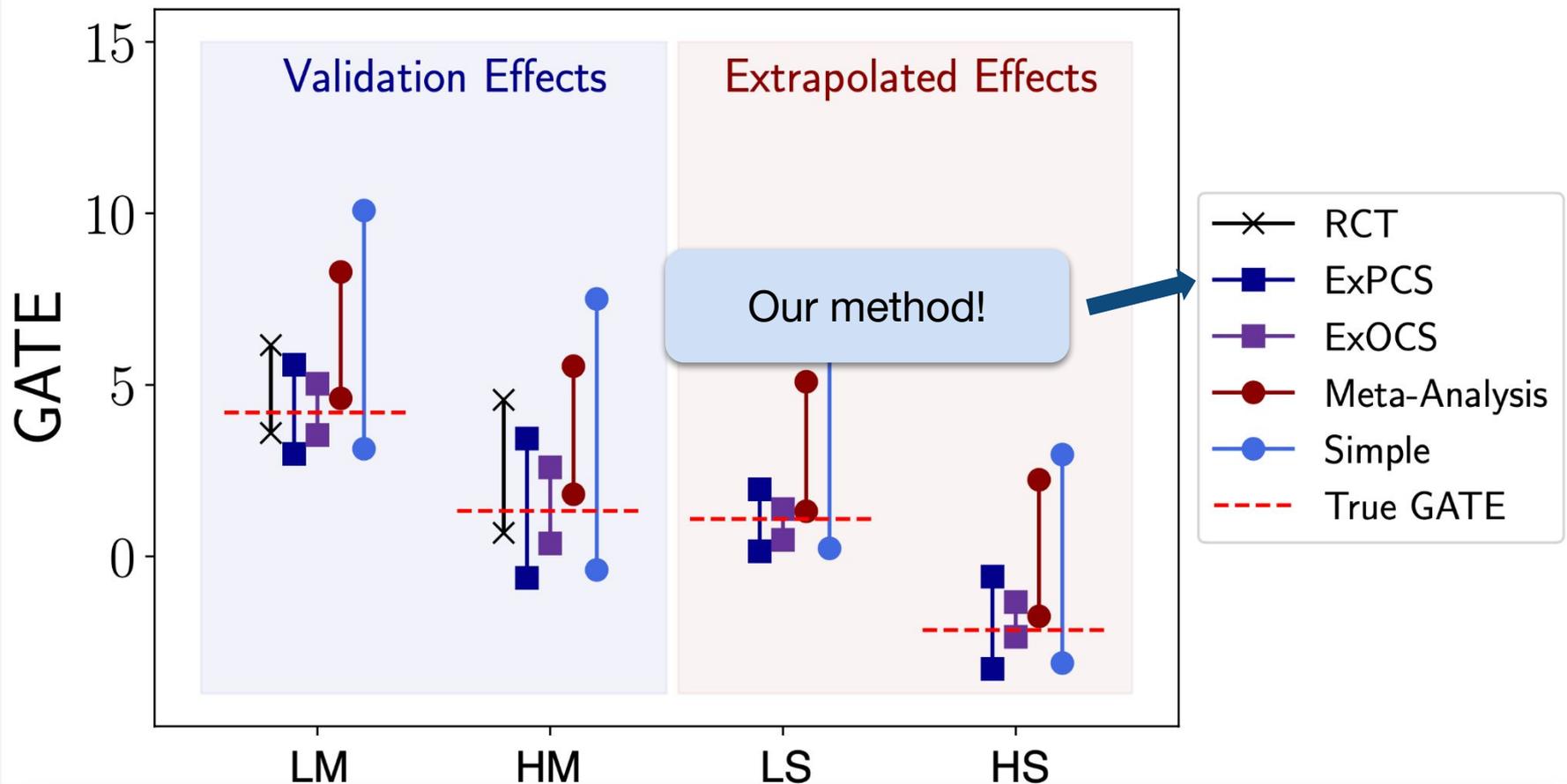
Pessimistic Combination
of Confidence Intervals

2

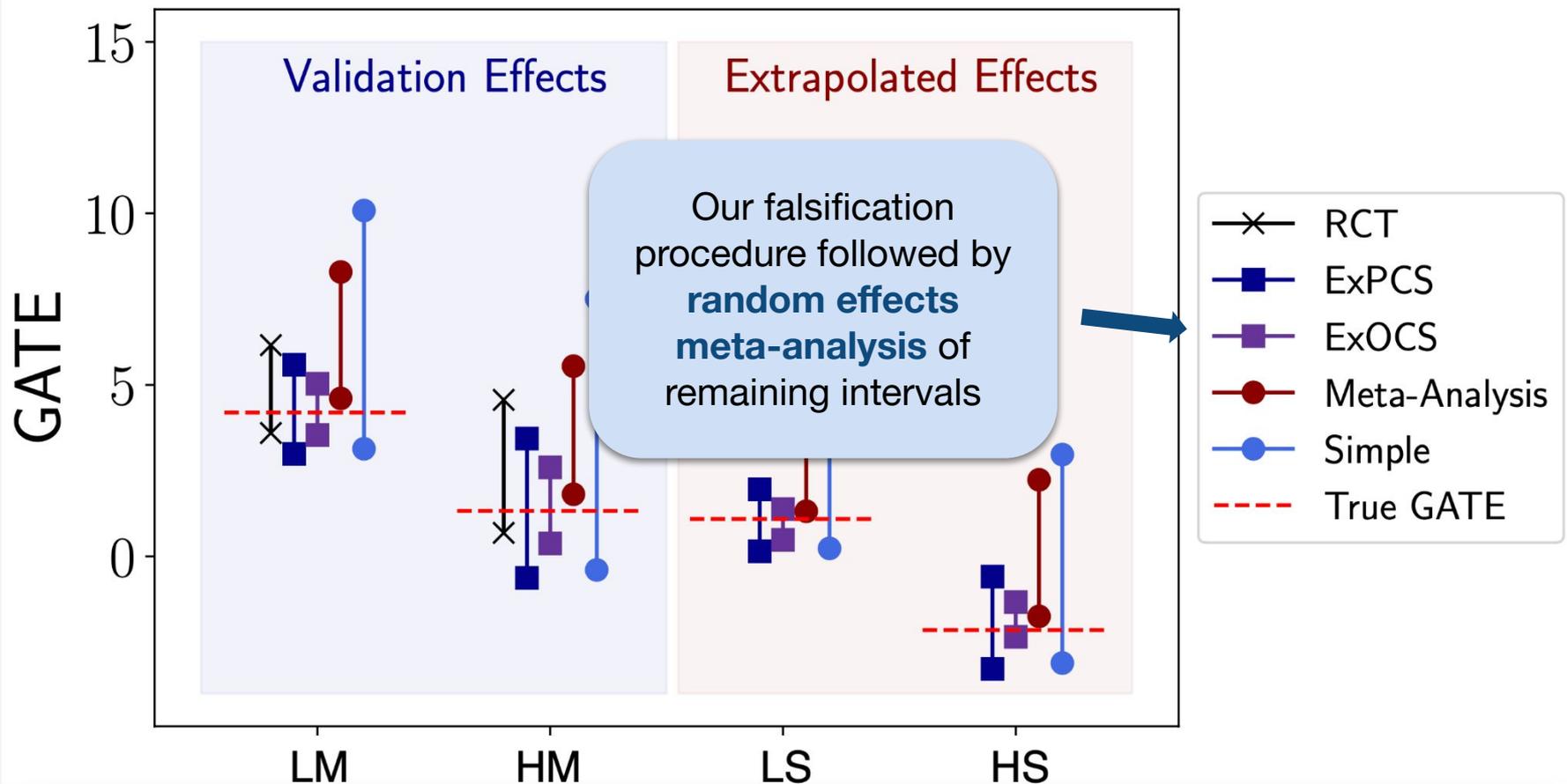
Pessimistic Combination of Confidence Intervals



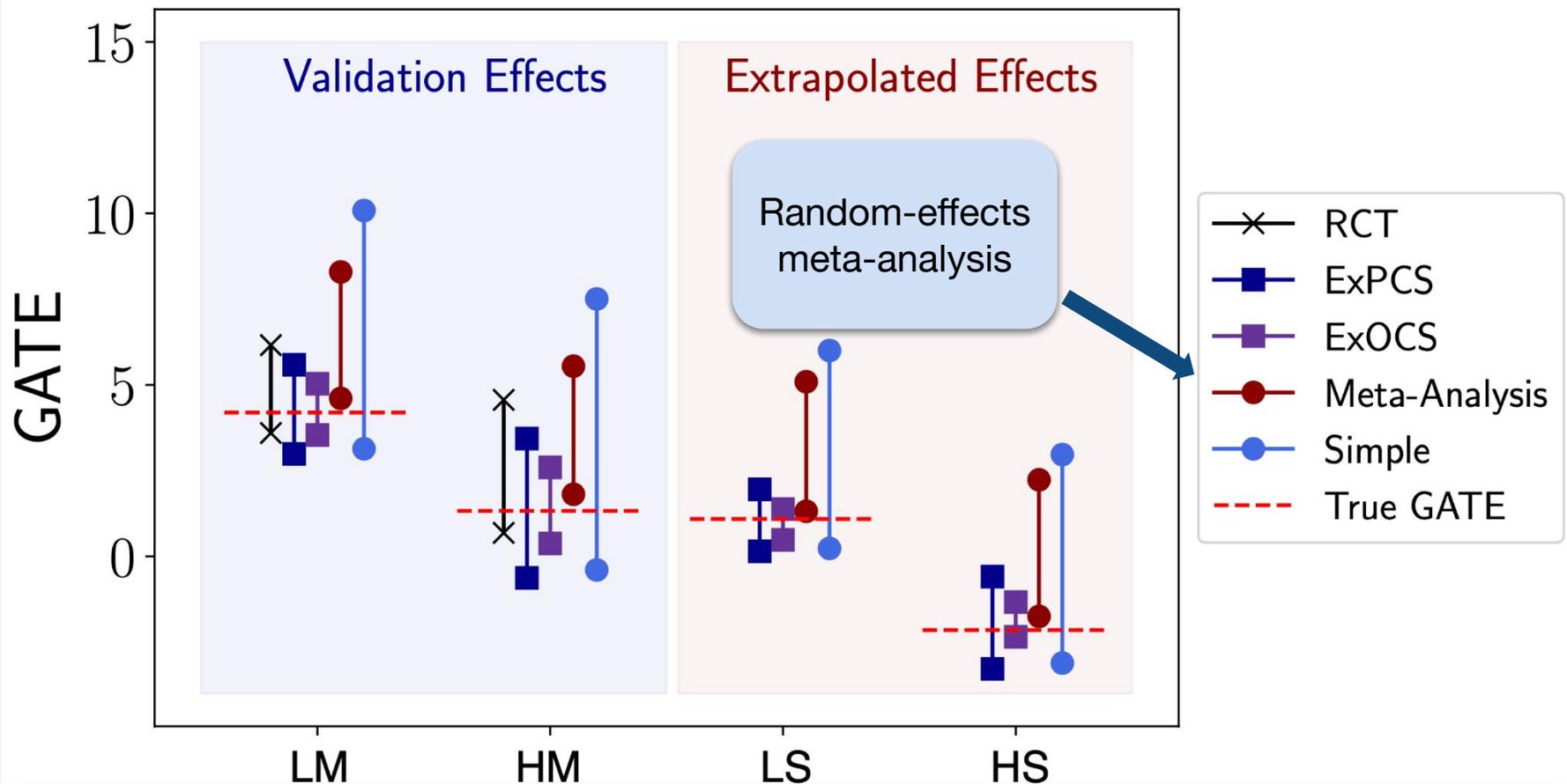
Results on Semi-Synthetic



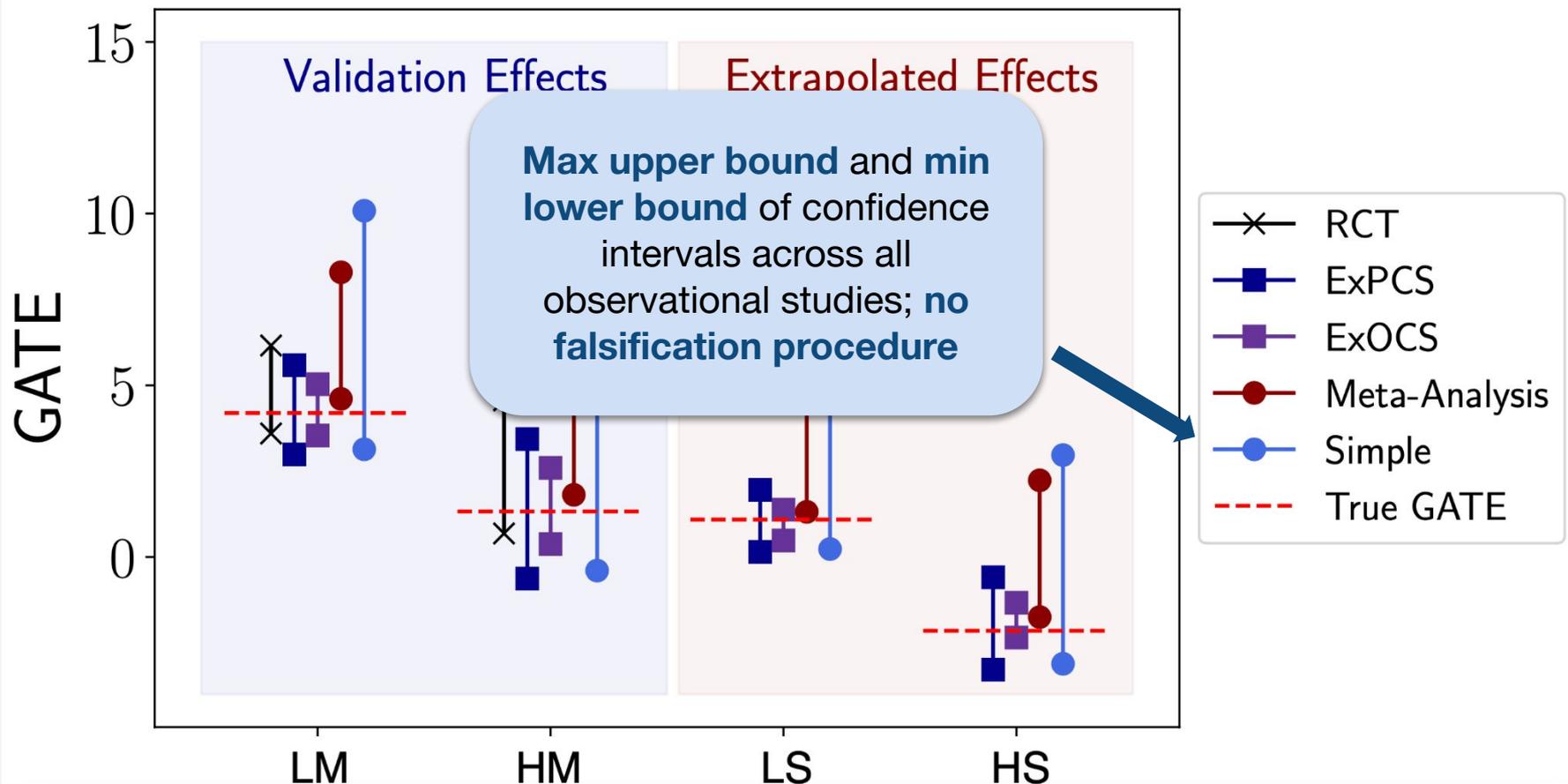
Results on Semi-Synthetic



Results on Semi-Synthetic

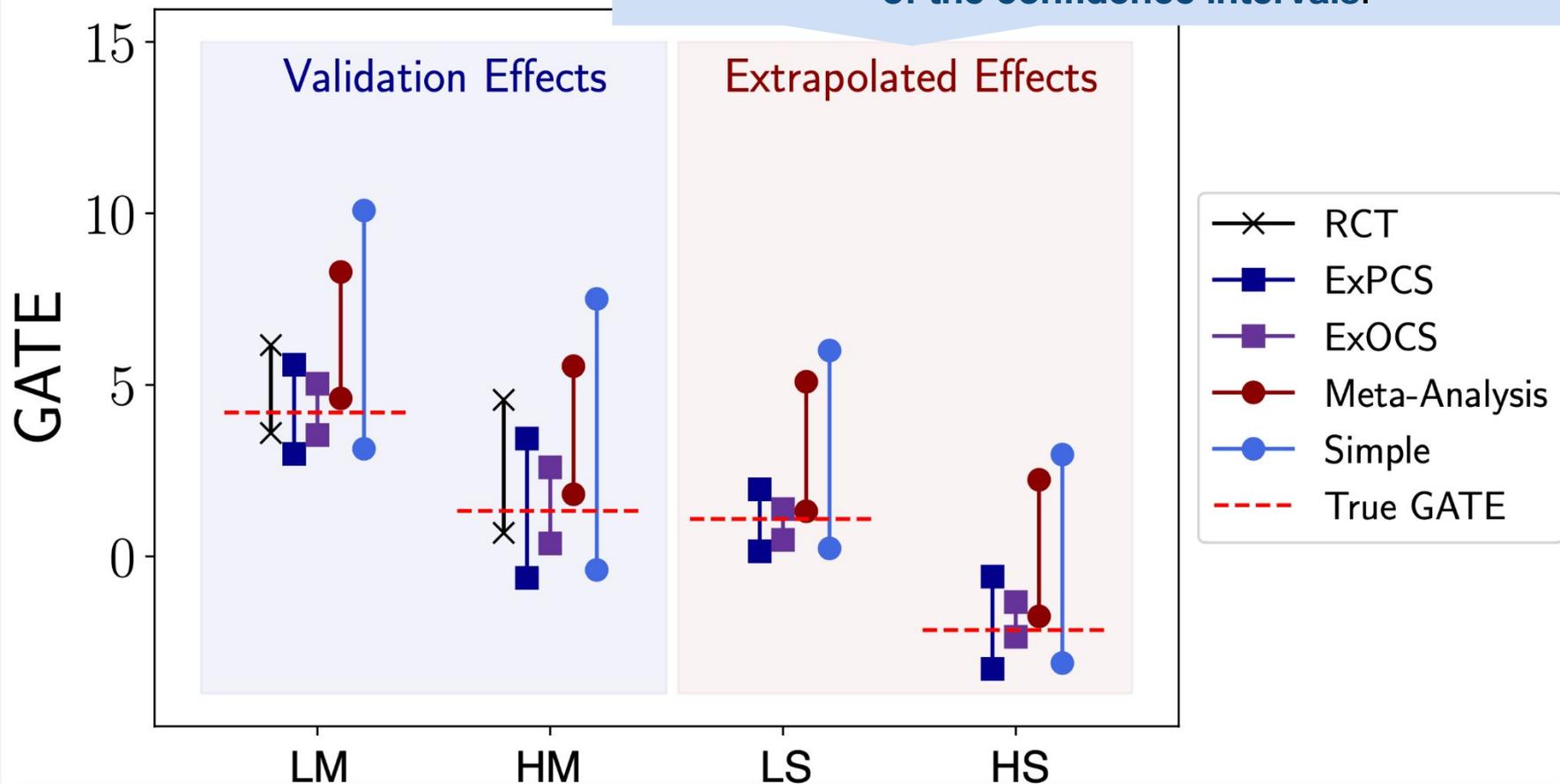


Results on Semi-Synthetic



Results on Semi-S

Compared to baselines, our approach has the best balance between **coverage of the true GATE** and **width of the confidence intervals**.



For more results and discussion, visit
us at poster ID 54677!

Thank you!

Results on Women's Health Initiative Data

	Coverage	Length	OS %
Simple	0.39	0.416	–
Meta-Analysis	0.03	0.260	–
ExOCS	0.28	0.058	–
ExPCS (ours)	0.45	0.081	0.99
Oracle	0.44	0.068	–

Table 1: Coverage, length, and unbiased OS % of ExPCS and baselines. ExPCS achieves comparable coverage to the oracle method with highly efficient intervals. Additionally, we do not reject the unbiased OS in 99% of the tasks.