Deep Generalized Schrödinger Bridge

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Diffusion models in solving **social science problems**

Deep Generalized Schrödinger Bridge

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Diffusion models in solving social science problems

Collective behavior of individual agents
interacting and evolving with a large population

e.g., population modeling, opinion dynamics, etc

Deep Generalized Schrödinger Bridge

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Diffusion models in solving social science problems

Collective behavior of ***individual agents***
interacting and evolving with a ***large population***

Hamilton-Jacobi-Bellman (HJB) PDE

$$-\frac{\partial u(x,t)}{\partial t} = \nabla u^\top f(x,\rho) - \frac{1}{2}\|\sigma \nabla u\|^2 + \frac{1}{2}\sigma^2 \Delta u + F(x,\rho)$$

value function — *base drift* — *diffusion* — *mean-field interaction* — *population density*

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Diffusion models in solving social science problems

Collective behavior of ***individual agents***
interacting and ***evolving*** with a ***large population***

Fokker-Planck (FP) PDE

$$\frac{\partial \rho(x,t)}{\partial t} = \nabla \cdot \left(\rho\left(\sigma^2 \nabla u - f(x,\rho)\right)\right) + \frac{1}{2}\sigma^2 \Delta \rho$$

population density — *diffusion* — *base drift*

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Diffusion models in solving Mean-Field Games

with distributional boundary constraints

$$-\frac{\partial u(x,t)}{\partial t} = \nabla u^\top f(x,\rho) - \frac{1}{2}\|\sigma \nabla u\|^2 + \frac{1}{2}\sigma^2 \Delta u + F(x,\rho)$$

$$\frac{\partial \rho(x,t)}{\partial t} = \nabla \cdot \left(\rho\left(\sigma^2 \nabla u - f(x,\rho)\right)\right) + \frac{1}{2}\sigma^2 \Delta \rho$$

$$\text{subject to } \rho(x,0) = \rho_0, \quad \rho(x,T) = \rho_T$$

⬇

$$\mathrm{d}X_t = \left[f(X_t, \rho(X_t,t)) - \sigma^2 \nabla u(X_t,t)\right]\mathrm{d}t + \sigma \mathrm{d}W_t$$

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Diffusion models in solving Mean-Field Games

→ Schrödinger Bridge

with distributional boundary constraints

Optimal transport

Entropy-regularized

$$-\frac{\partial u(x,t)}{\partial t} = \nabla u^\top f(x,\rho) - \frac{1}{2}\|\sigma \nabla u\|^2 + \frac{1}{2}\sigma^2 \Delta u + F(x,\rho)$$

$$\frac{\partial \rho(x,t)}{\partial t} = \nabla \cdot \left(\rho\left(\sigma^2 \nabla u - f(x,\rho)\right)\right) + \frac{1}{2}\sigma^2 \Delta \rho$$

$$\text{subject to } \rho(x,0) = \rho_0, \quad \rho(x,T) = \rho_T$$

$$\mathrm{d}X_t = \left[f(X_t, \rho(X_t,t)) - \sigma^2 \nabla u(X_t,t)\right]\mathrm{d}t + \sigma \mathrm{d}W_t$$

Deep Generalized Schrödinger Bridge

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Schrödinger Bridge in solving **Mean-Field Games**

generalized to mean-field structure

with distributional boundary constraints

Forward-Backward SDEs ← *Hopf-Cole + Nonlinear Feynman-Kac* – – – **HJB-FP PDEs**

Mean-field SDE

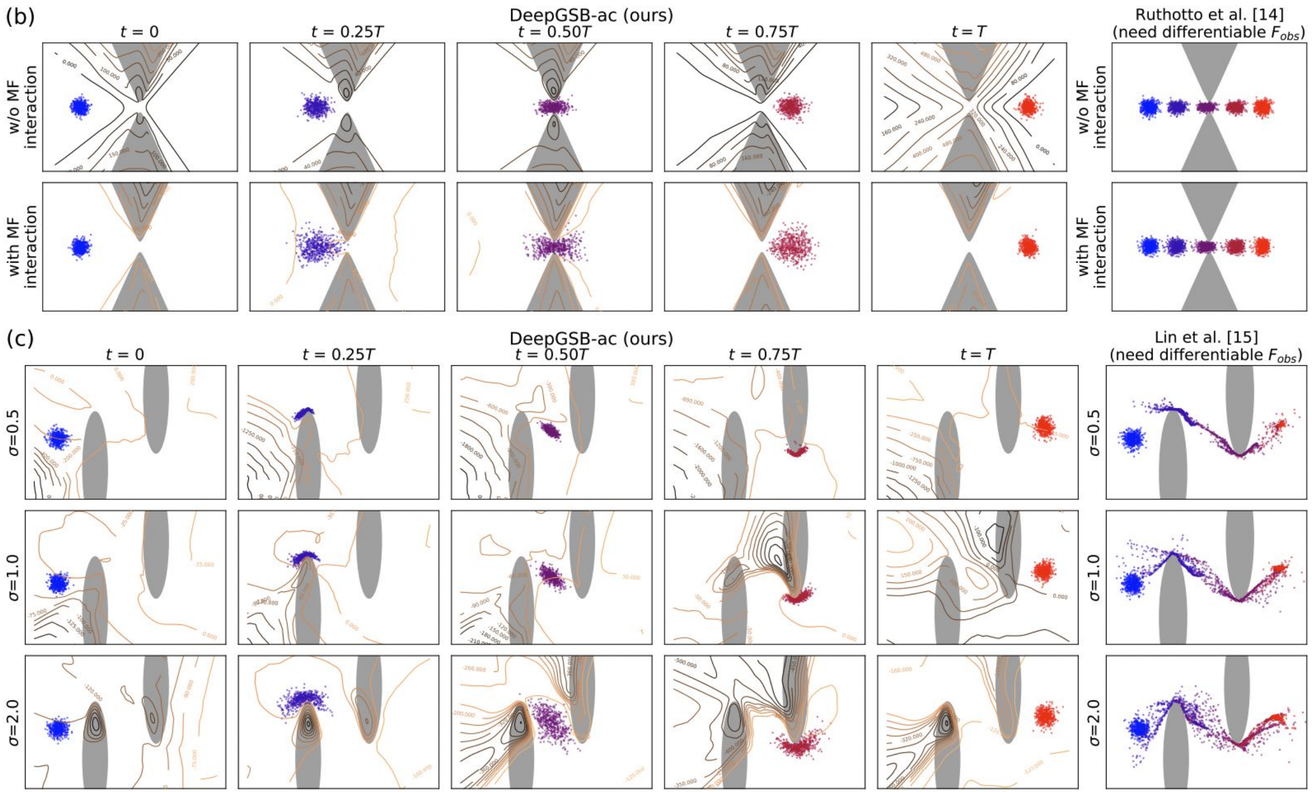
Stochastic Bellman

Necessary and sufficient conditions *(proven in paper)*

Training objective = log-likelihood + Temporal Difference loss

Chen & Liu* et al., "Likelihood training of SB", ICLR 2022*

Crowd Navigation Mean-Field Game



Code

Opinion Depolarization Mean-Field Game

Polarize base dynamic