



# Theseus

A library for differentiable nonlinear optimization



Luis  
Pineda



Taosha  
Fan



Maurizio  
Monge



Shobha  
Venkataraman



Paloma  
Sodhi



Ricky T. Q.  
Chen



Joseph  
Ortiz



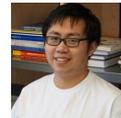
Daniel  
DeTone



Austin  
Wang



Stuart  
Anderson



Jing  
Dong



Brandon  
Amos



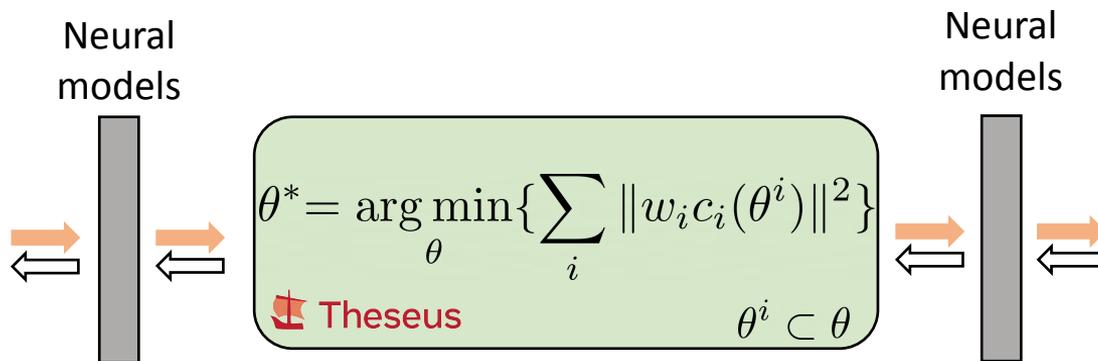
Mustafa  
Mukadam

```
pip install theseus-ai
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<https://sites.google.com/view/theseus-ai>

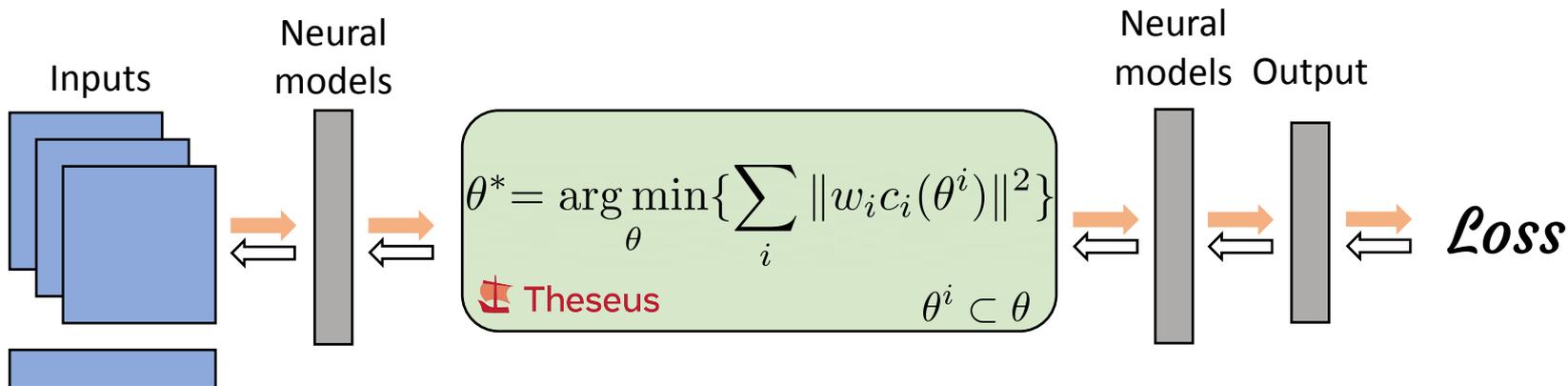
# What is Theseus?



Theseus is a  
nonlinear optimization layers in PyTorch

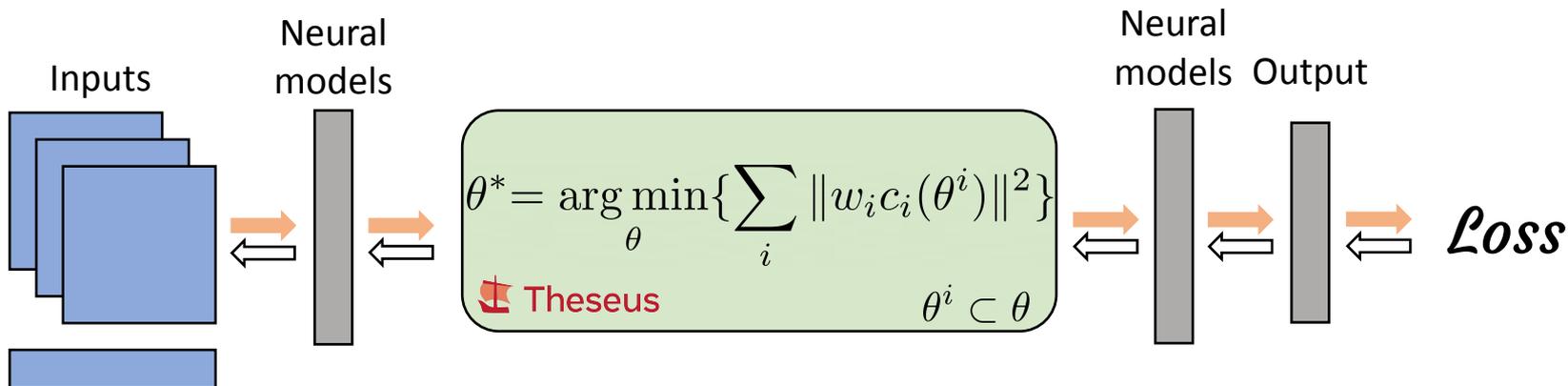
library for building custom

# What is Theseus?



Theseus is a **nonlinear optimization layers** library for building custom end-to-end differentiable architectures in PyTorch to support constructing

# What is Theseus?



Theseus is an **efficient application-agnostic** library for building custom **nonlinear optimization layers** in PyTorch to support constructing various problems in robotics and vision as end-to-end differentiable architectures

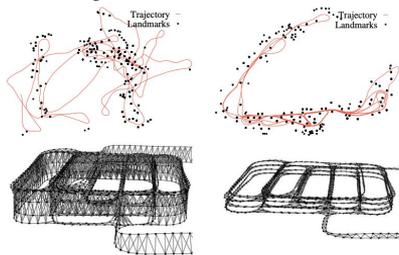
# Structure-from-Motion Revisited

Johannes L. Schönberger<sup>1,2\*</sup>, Jan-Michael Frahm<sup>1</sup>



## $g^2o$ : A General Framework for Graph Optimization

Rainer Kümmeler   Giorgio Grisetti   Hauke Strasdat   Kurt Konolige   Wolfram Burgard



# Tracking many objects with many sensors

Hanna Pasula and Stuart Russell   Michael Ostland and Ya'acov Ritov\*

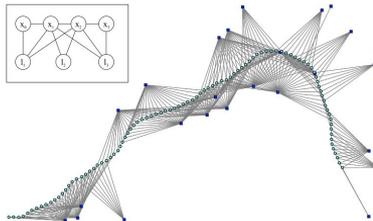
## Generalized-ICP

Aleksandr V. Segal   Dirk Haehnel   Sebastian Thrun

## Square Root SAM

Simultaneous Localization and Mapping  
via Square Root Information Smoothing

Frank Dellaert and Michael Kaess

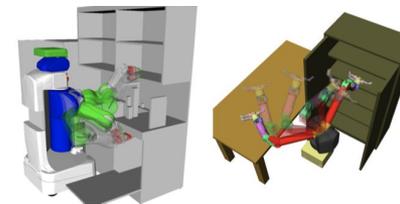


# Recovering 3D Shape and Motion from Image Streams using Non-Linear Least Squares

Richard Szeliski and Sing Bing Kang

## Continuous-time Gaussian process motion planning via probabilistic inference

Mustafa Mukadam\*, Jing Dong\*, Xinyan Yan, Frank Dellaert and Byron Boots

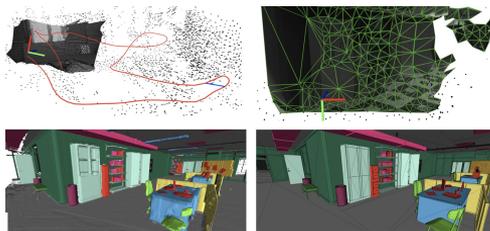


## Bundle Adjustment — A Modern Synthesis

Bill Triggs<sup>1</sup>, Philip McLauchlan<sup>2</sup>, Richard Hartley<sup>3</sup> and Andrew Fitzgibbon<sup>4</sup>

# Kimera: an Open-Source Library for Real-Time Metric-Semantic Localization and Mapping

Antoni Rosinol, Marcus Abate, Yun Chang, Luca Carlone

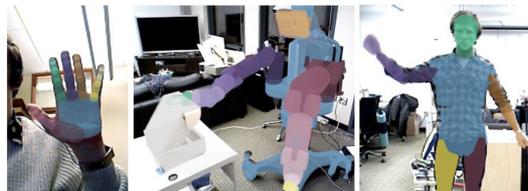


## A Family of Iterative Gauss-Newton Shooting Methods for Nonlinear Optimal Control

Markus Gifthalder<sup>1</sup>, Michael Neunert<sup>1</sup>, Markus Stäuble<sup>1</sup>, Jonas Buchli<sup>1</sup> and Moritz Diehl<sup>2</sup>

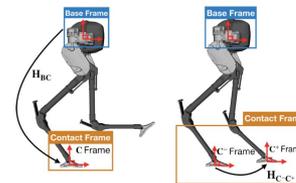
# DART: Dense Articulated Real-Time Tracking

Tanner Schmidt, Richard Newcombe, Dieter Fox



## Hybrid Contact Preintegration for Visual-Inertial-Contact State Estimation Using Factor Graphs

Ross Hartley, Maani Ghaffari Jadidi, Lu Gan, Jiunn-Kai Huang, Jessy W. Grizzle, and Ryan M. Eustice



# Structure-from-Motion Revisited

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Recovering 3D Shape and Motion from Image Streams using Non-Linear Least Squares

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## Generalized-ICP

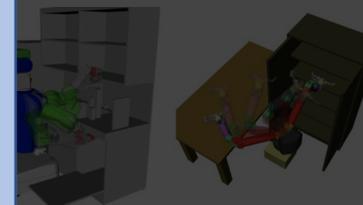
Aleksandr V. Segal

Dirk Haehnel

Sebastian Thrun

Continuous-time Gaussian process Mapping via probabilistic

..., Jing Dong\*, Xinyan Yan, Frank Dellaert and Byron Boots



Adjustment — A Modern Synthesis

McLauchlan<sup>2</sup>, Richard Hartley<sup>3</sup> and Andrew Fitzgibbon<sup>4</sup>

Contact Preintegration for Visual-Inertial-Contact State Estimation Using Factor Graphs

..., Ghaffari Jadidi, Lu Gan, Jiunn-Kai Huang, Jessy W. Grizzle, and Ryan M. Eustice



## SLAM

- Bundle adjustment
- Structure from motion
- Tracking and estimation
- Motion planning
- Optimal control

...

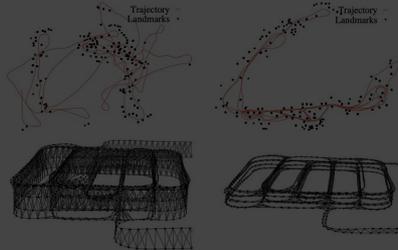


Johannes L. Schönberger<sup>1,2\*</sup>, Jan-Michael Frahm<sup>1</sup>



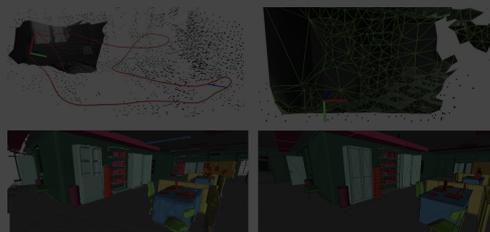
## g<sup>2</sup>o: A General Framework for Graph Opti

Rainer Kümmerle Giorgio Grisetti Hauke Strasdat Kurt Konolige

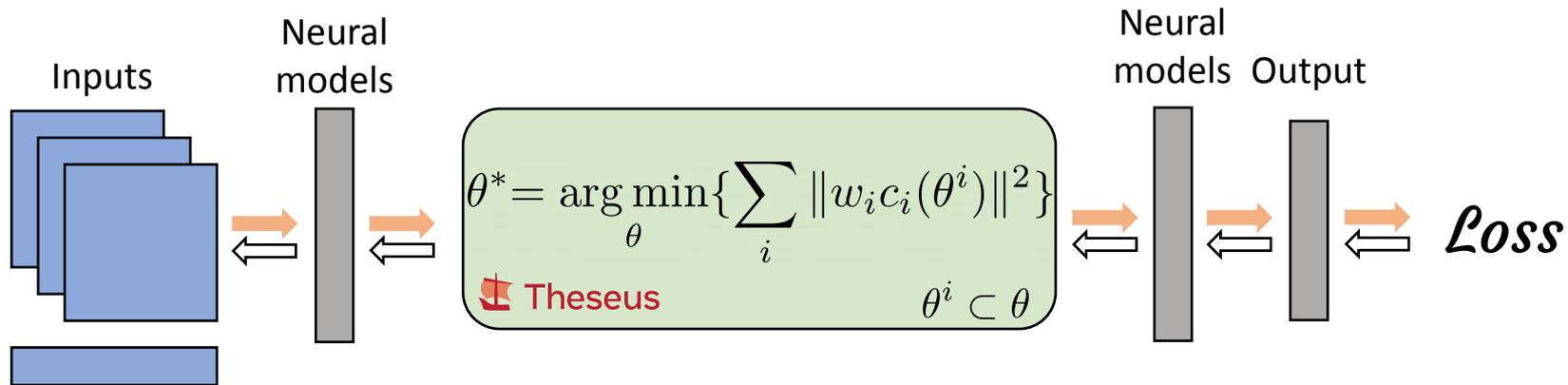


## Kimera: an Open-Source Library for Real-T Metric-Semantic Localization and Mapping

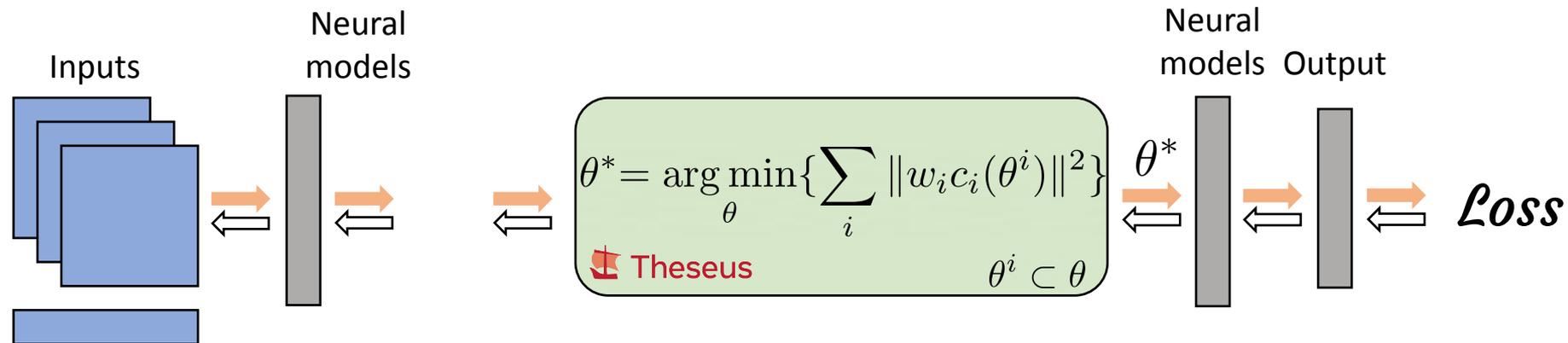
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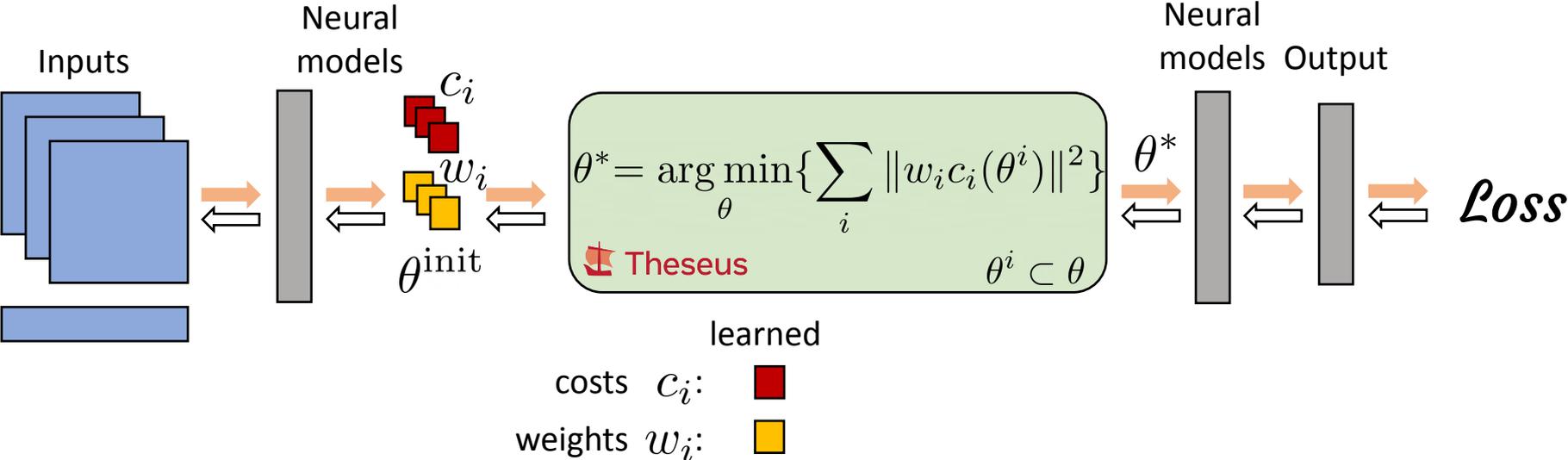
# What is Theseus?



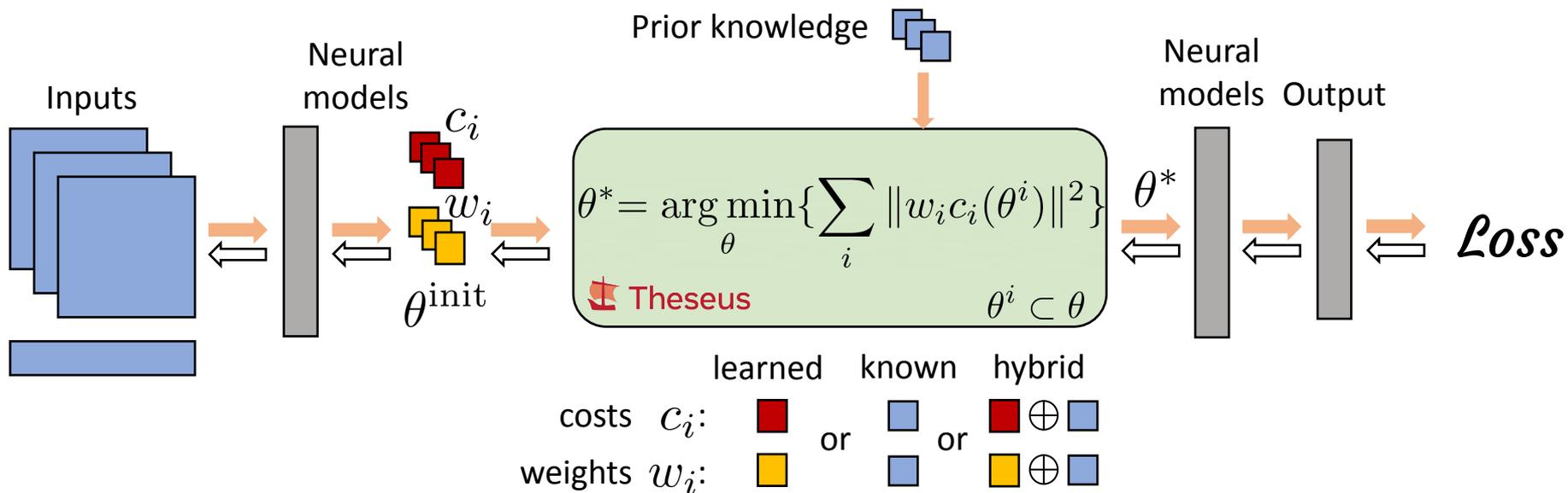
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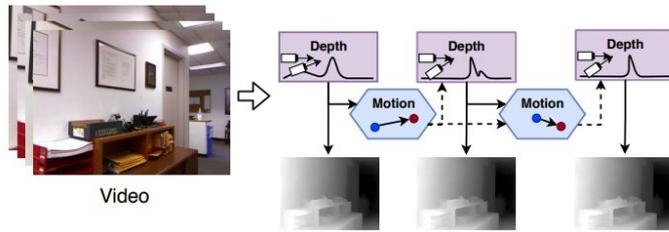
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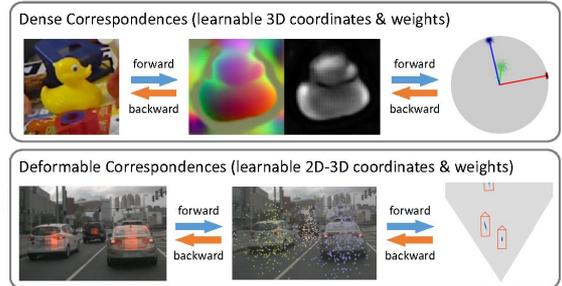
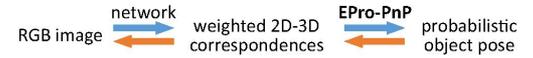
Taking a Deeper Look at the Inverse Compositional Algorithm

Zhaoyang Lv<sup>1,2</sup> Frank Dellaert<sup>1</sup> James M. Rehg<sup>1</sup> Andreas Geiger<sup>2</sup>



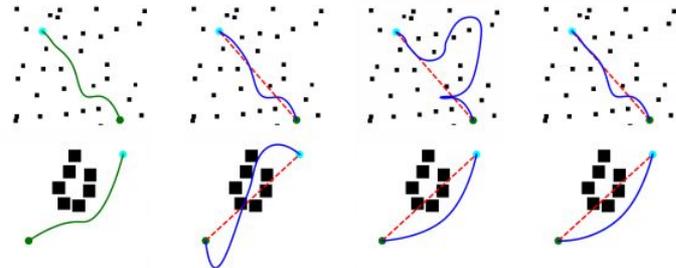
DEEPV2D: VIDEO TO DEPTH WITH DIFFERENTIABLE STRUCTURE FROM MOTION

Zachary Teed Jia Deng



EPro-PnP: Generalized End-to-End Probabilistic Perspective-n-Points for Monocular Object Pose Estimation

Hansheng Chen<sup>1,2,\*</sup> Pichao Wang<sup>2,†</sup> Fan Wang<sup>2</sup> Wei Tian<sup>1,†</sup> Lu Xiong<sup>1</sup> Hao Li<sup>2</sup>  
<sup>1</sup>School of Automotive Studies, Tongji University <sup>2</sup>Alibaba Group



Differentiable Gaussian Process Motion Planning

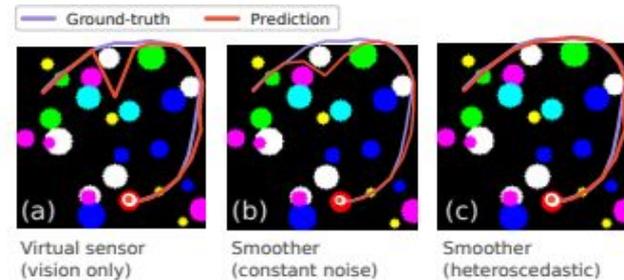
Mohak Bhardwaj<sup>1</sup>, Byron Boots<sup>1</sup>, and Mustafa Mukadam<sup>2</sup>



$\nabla$ SLAM: Automatically differentiable SLAM

<https://gradslam.github.io>

Krishna Murthy J.<sup>\*1,2,3</sup>, Soroush Saryzadi<sup>\*4</sup>, Ganesh Iyer<sup>5</sup>, and Liam Paull<sup>†1,2,3,6</sup>

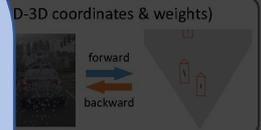
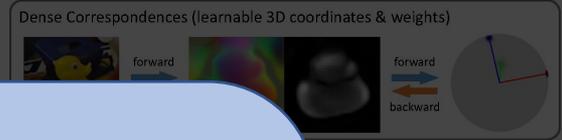


Differentiable Factor Graph Optimization for Learning Smoothers

Brent Yi<sup>1</sup>, Michelle A. Lee<sup>1</sup>, Alina Kloss<sup>2</sup>, Roberto Martín-Martín<sup>1</sup>, and Jeannette Bohg<sup>1</sup>

## The literature is fragmented

- Implementations are application specific
- Limited batching and GPU support
- Do not leverage sparsity
- Backprop only via unrolling



### Probabilistic Perspective-n-Points Pose Estimation

Wei Tian<sup>1,†</sup> Lu Xiong<sup>1</sup> Hao Li<sup>2</sup>  
University <sup>2</sup>Alibaba Group

Virtual sensor  
(vision only)

Smoother  
(constant noise)

Smoother  
(heteroscedastic)

### Differentiable Gaussian Process Motion Planning

Mohak Bhardwaj<sup>1</sup>, Byron Boots<sup>1</sup>, and Mustafa Mukadam<sup>2</sup>

### $\nabla$ SLAM: Automatically differentiable SLAM

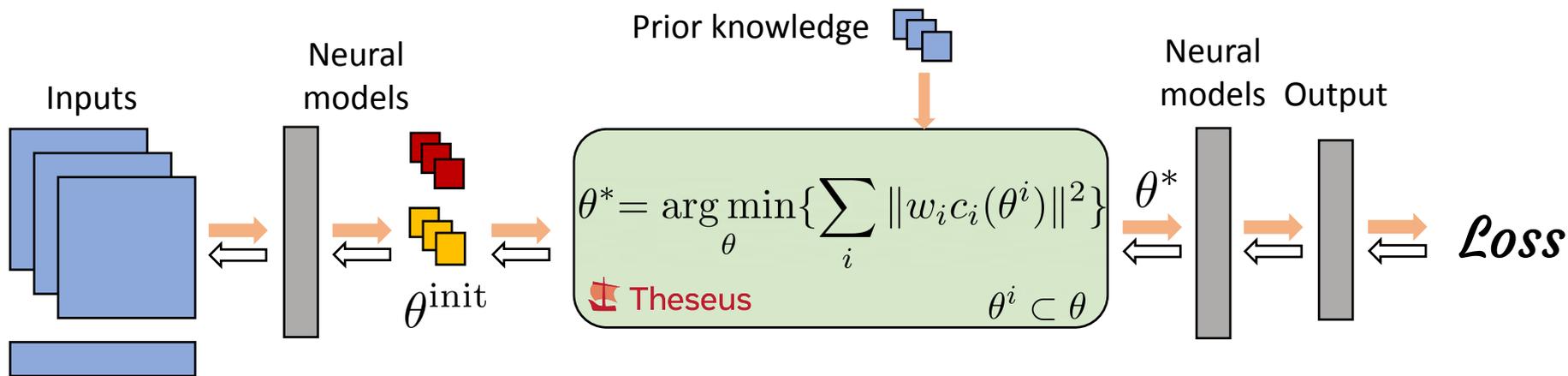
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### Differentiable Factor Graph Optimization for Learning Smoother

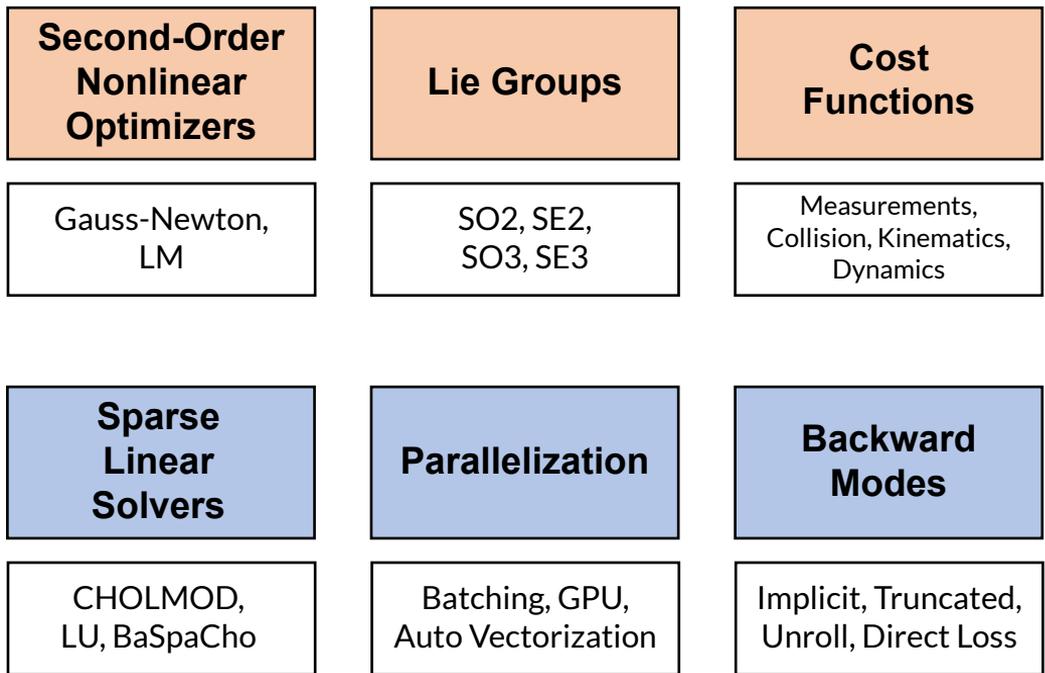
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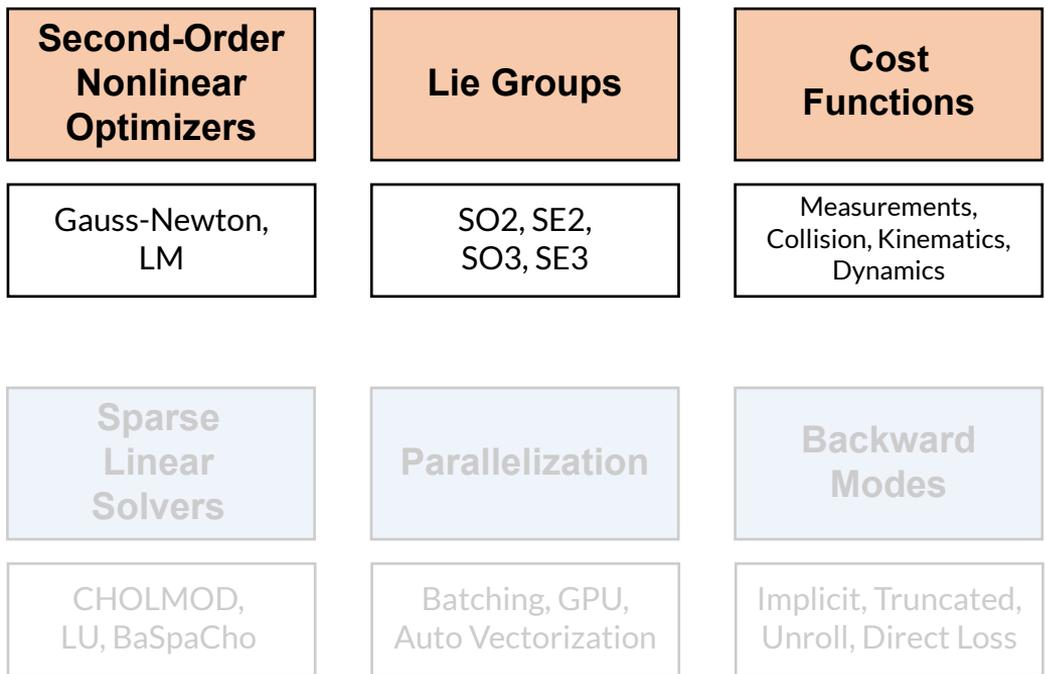
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## Application Agnostic



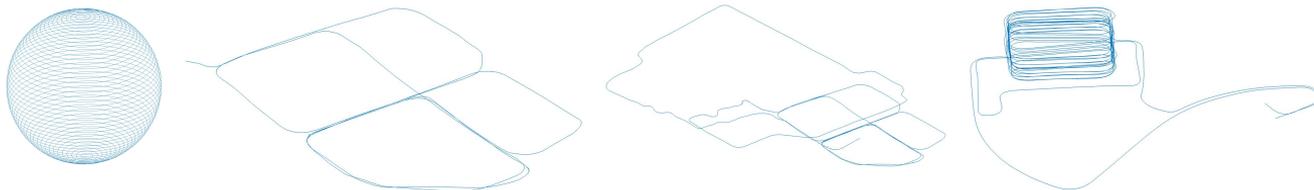
## Efficient

## Application Agnostic

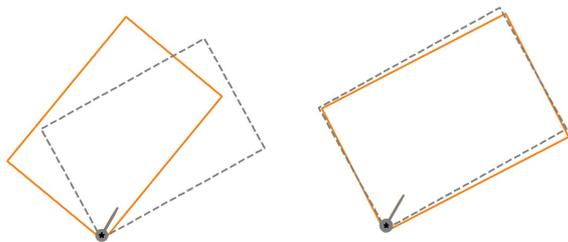


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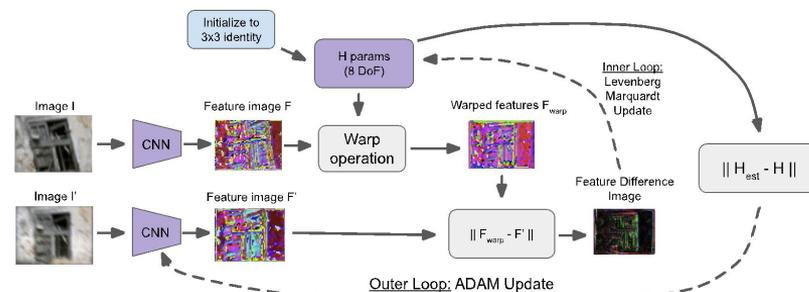
# Pose graph optimization



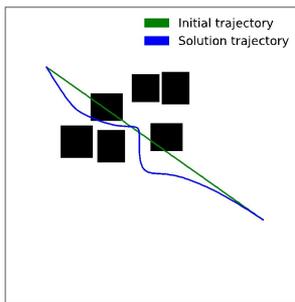
## Tactile state estimation



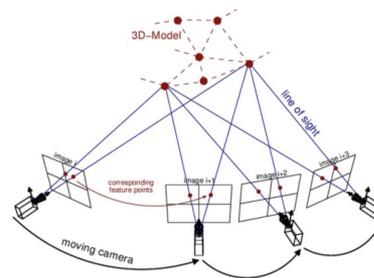
## Homography estimation



## Motion Planning



## Bundle adjustment



## Application Agnostic

**Second-Order  
Nonlinear  
Optimizers**

Gauss-Newton,  
LM

**Lie Groups**

SO2, SE2,  
SO3, SE3

**Cost  
Functions**

Measurements,  
Collision, Kinematics,  
Dynamics

## Efficient

**Sparse  
Linear  
Solvers**

CHOLMOD,  
LU, BaSpaCho

**Parallelization**

Batching, GPU,  
Auto Vectorization

**Backward  
Modes**

Implicit, Truncated,  
Unroll, Direct Loss

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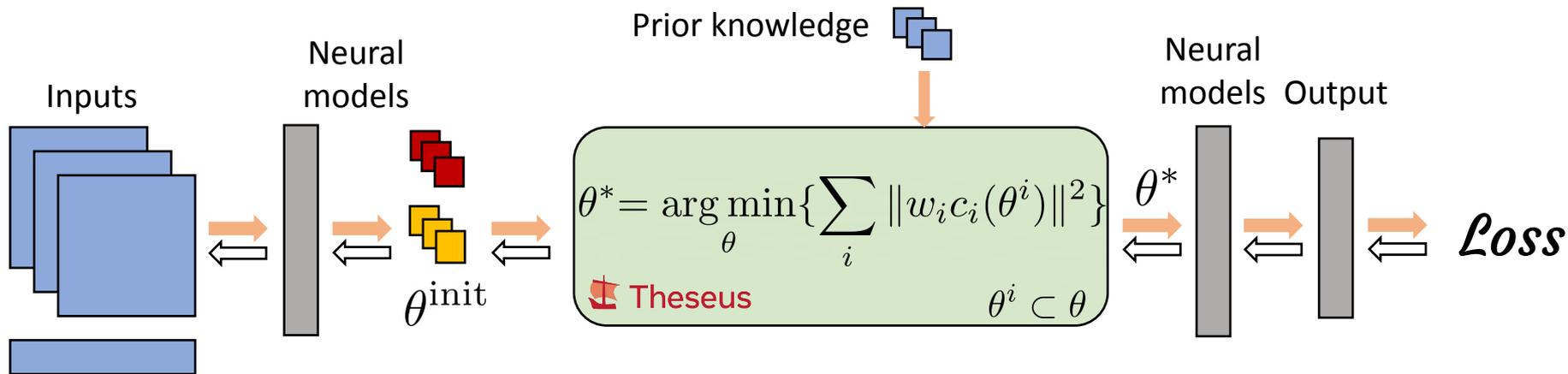
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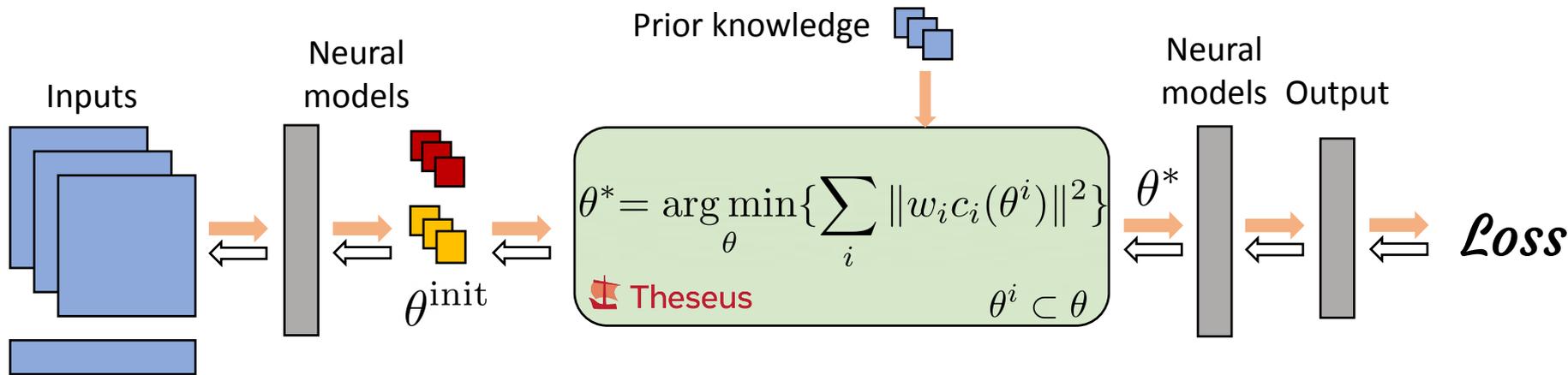


Step 1: Linearize  $A\delta\theta = b$

Step 2: Linear solve  $\delta\theta$

Step 3: Update  $\theta \leftarrow \theta + \delta\theta$

repeat

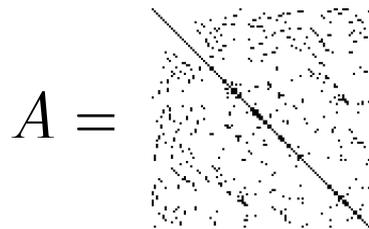


Step 1: Linearize  $A\delta\theta = b$

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repeat



$A =$   
torch.solve() treats A  
as a dense matrix

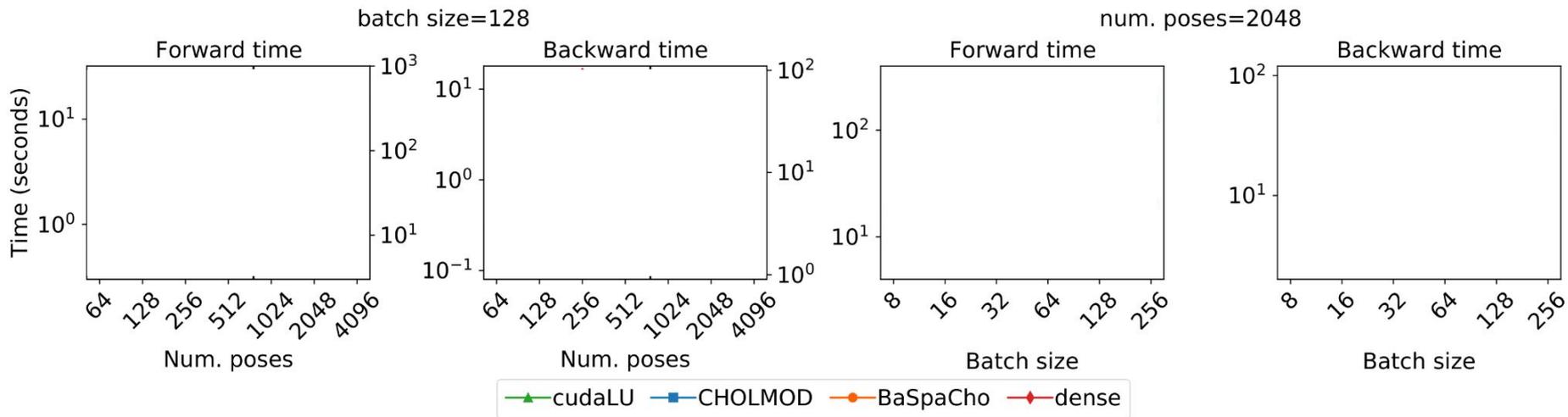
# Sparse vs Dense solvers

## Pose Graph Optimization

—▲— cudaLU —■— CHOLMOD —●— BaSpaCho —◆— dense

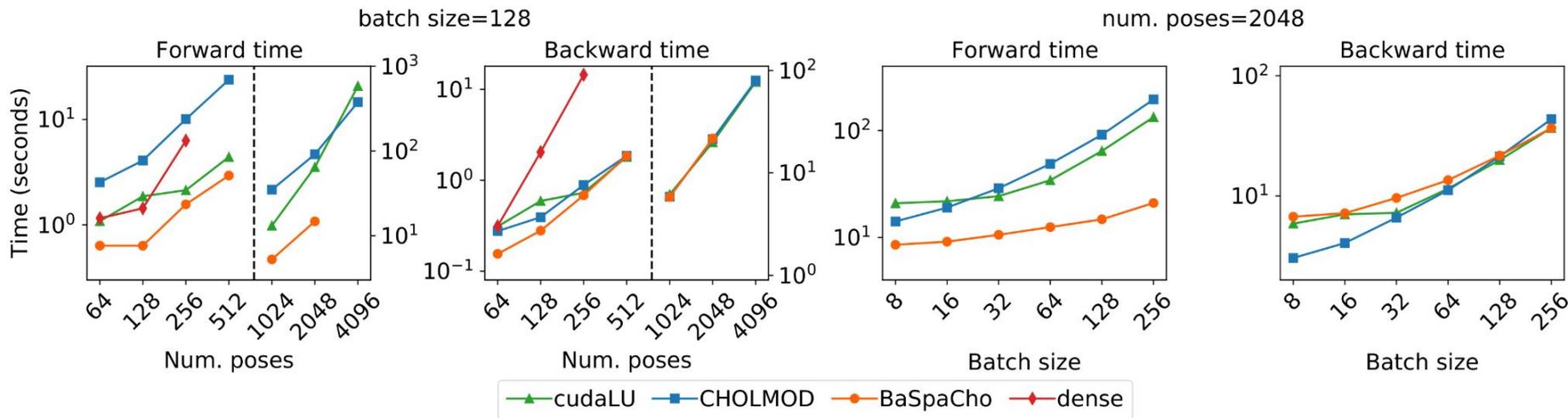
# Sparse vs Dense solvers

## Pose Graph Optimization



# Sparse vs Dense solvers

## Pose Graph Optimization



- Scales to 256 batch x 4096 poses on a standard GPU
- Dense solvers are slow and run out of memory

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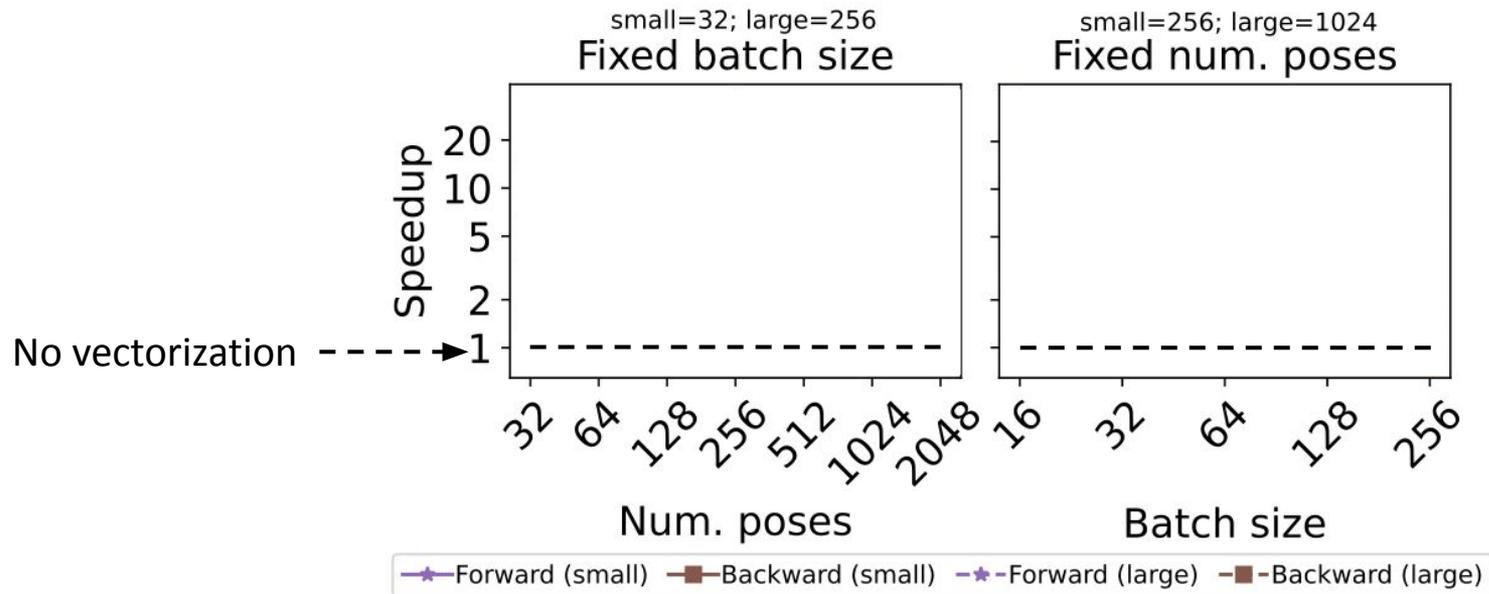
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Batching, GPU,  
Auto Vectorization

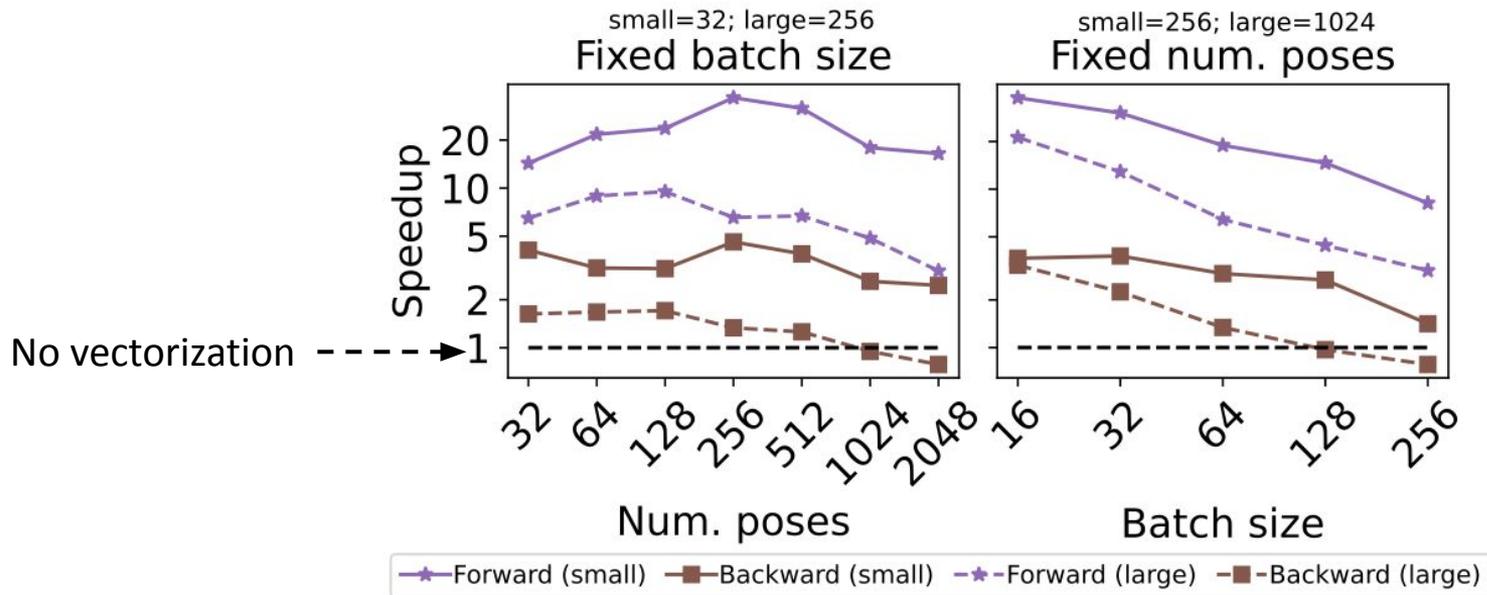
**Backward  
Modes**

Implicit, Truncated,  
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# Automatic vectorization



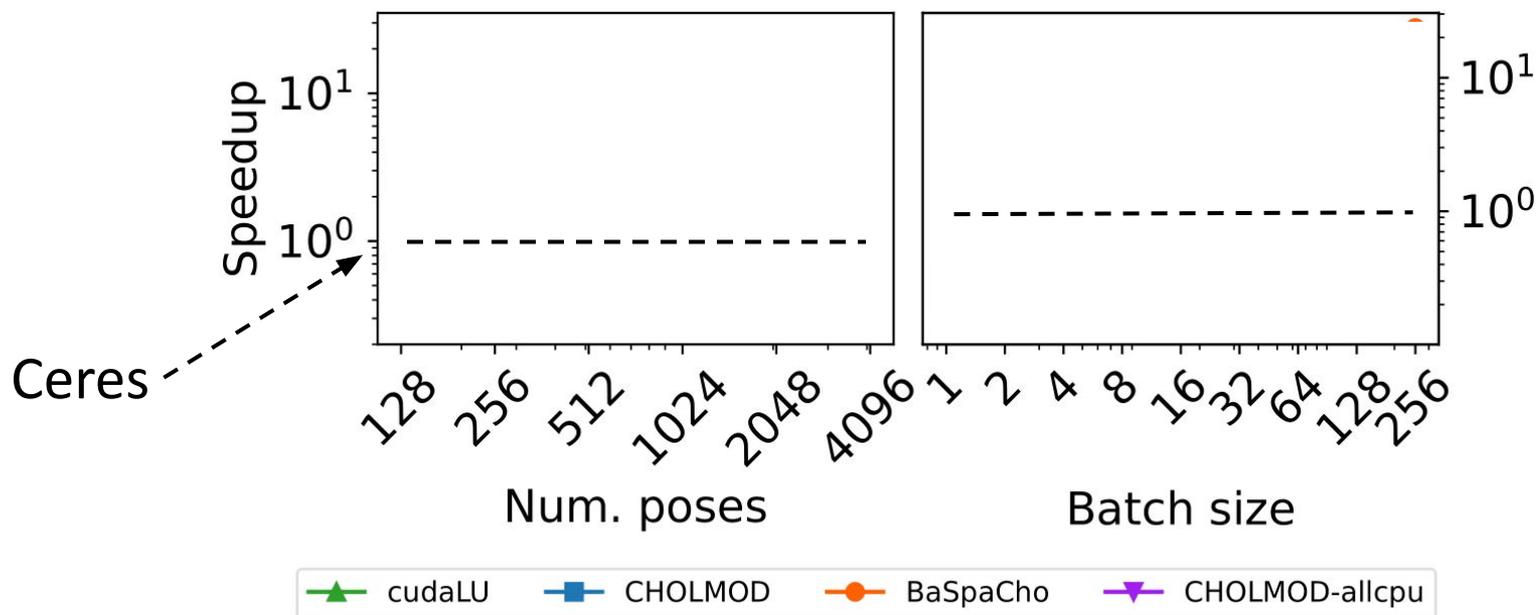
# Automatic vectorization



- Up to an order of magnitude speed-up

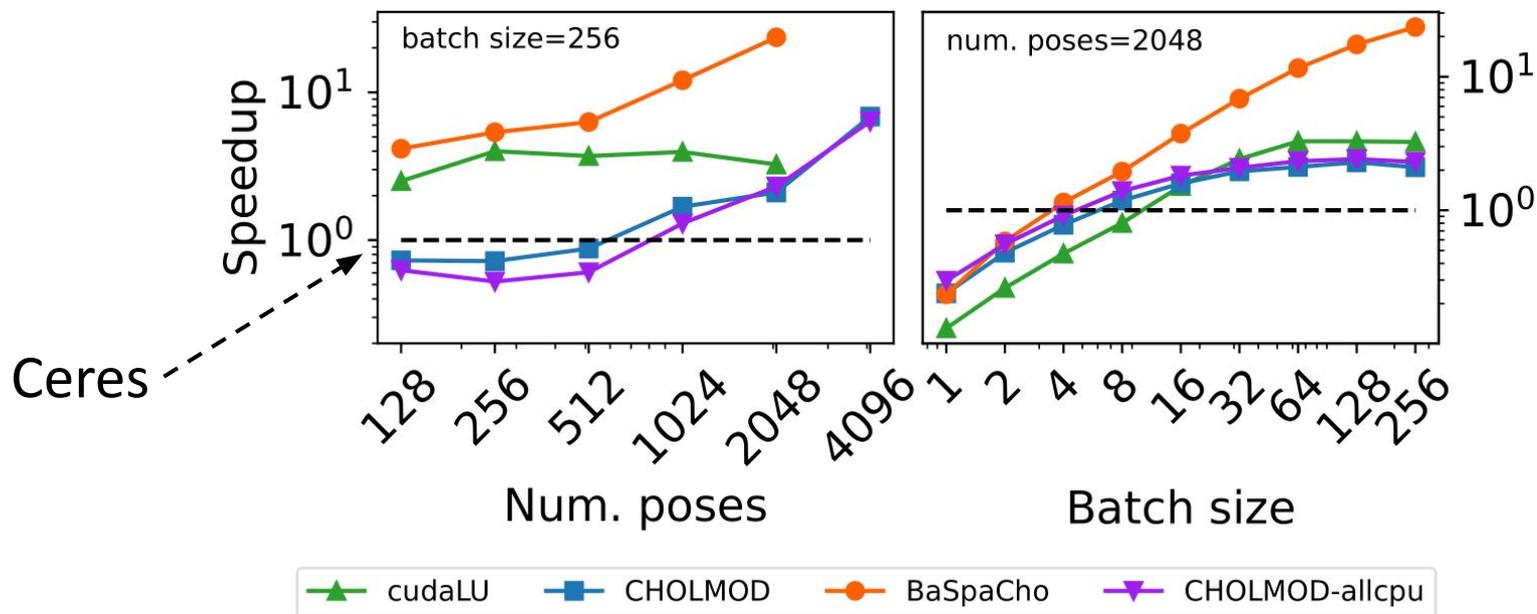
# Theseus forward vs Ceres

## Pose Graph Optimization



# Theseus forward vs Ceres

## Pose Graph Optimization



- Up to 20x speed-up over Ceres

## Application Agnostic

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SO3, SE3

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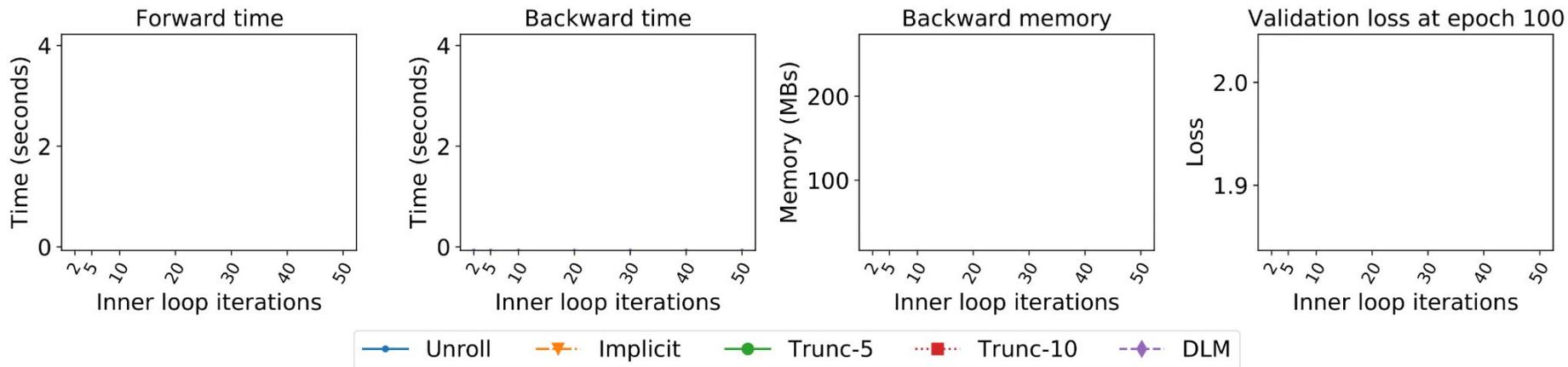
# Backward modes

## Tactile state estimation



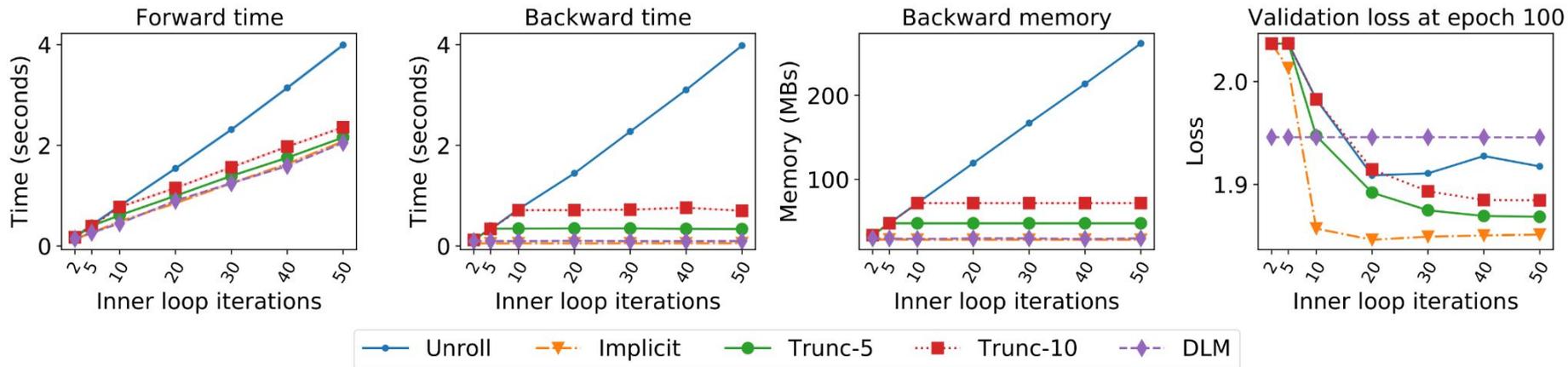
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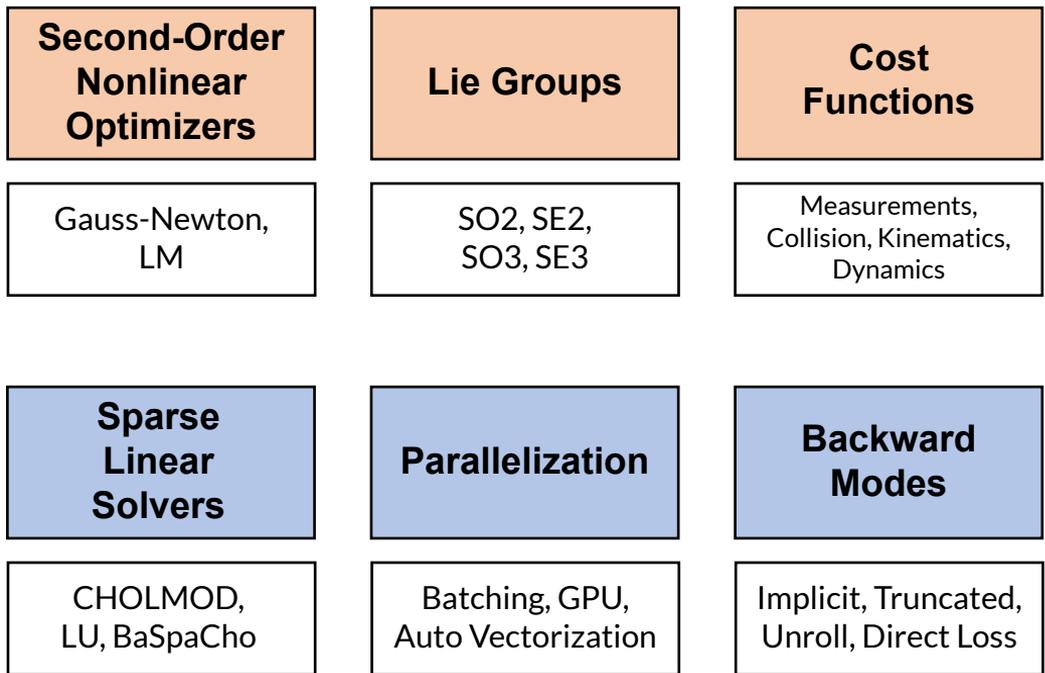
# Backward modes

## Tactile state estimation



- Implicit needs only constant time and memory
- Implicit gives better gradients
  - Even with few inner loop iterations

## Application Agnostic



## Efficient



# Theseus

A library for differentiable nonlinear optimization



Luis  
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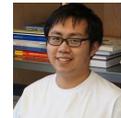
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<https://sites.google.com/view/theseus-ai>