



# SPA: A Graph Spectral Alignment Perspective for Domain Adaptation

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# Background

- Domain adaptation (DA) aims to **transfer** knowledge from label-rich source domains to label-scare target domains where domain shift exists.
- ⇒ learn **domain-invariant** feature representations
- ⇒ moment matching methods
  - ⇒ adversarial learning methods

# Background

- The most essential challenge of DA is how to find a suitable utilization of **intra-domain** information and **inter-domain** information to properly align target samples.
  - ⇒ discriminate data samples of different categories within a domain to the greatest extent possible (**intra-domain, discriminability**)
  - ⇒ learn transferable features across domains with the existence of domain shift (**inter-domain, transferability**)

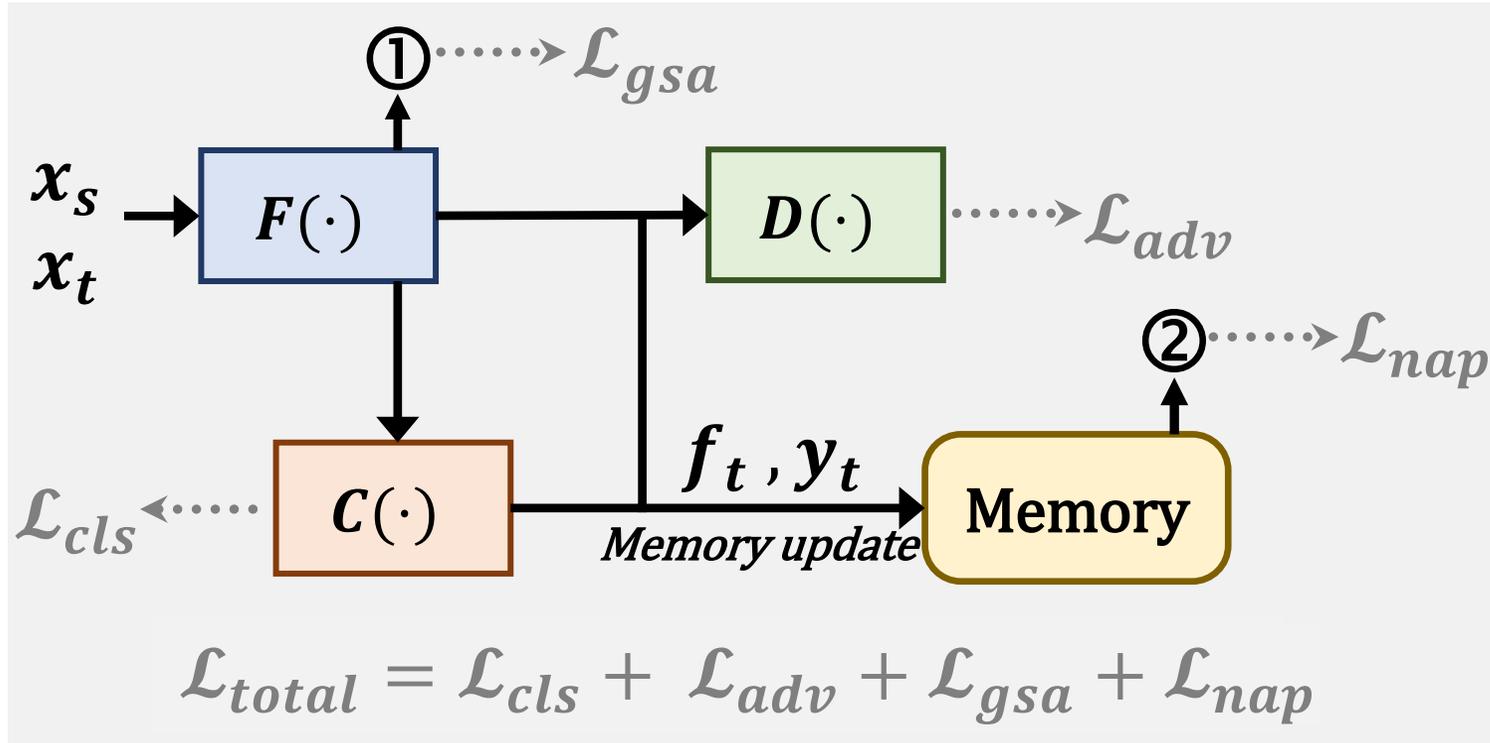
# Background

- Adversarial learning methods implicitly mitigate the domain shift by driving the feature extractor to capture indistinguishable features and fool the domain classifier.
- **Remarkable transferability** of some adversarial learning DA models is enhanced at the expense of **worse discriminability**.

⇒ **Trade-off** between transferability and discriminability

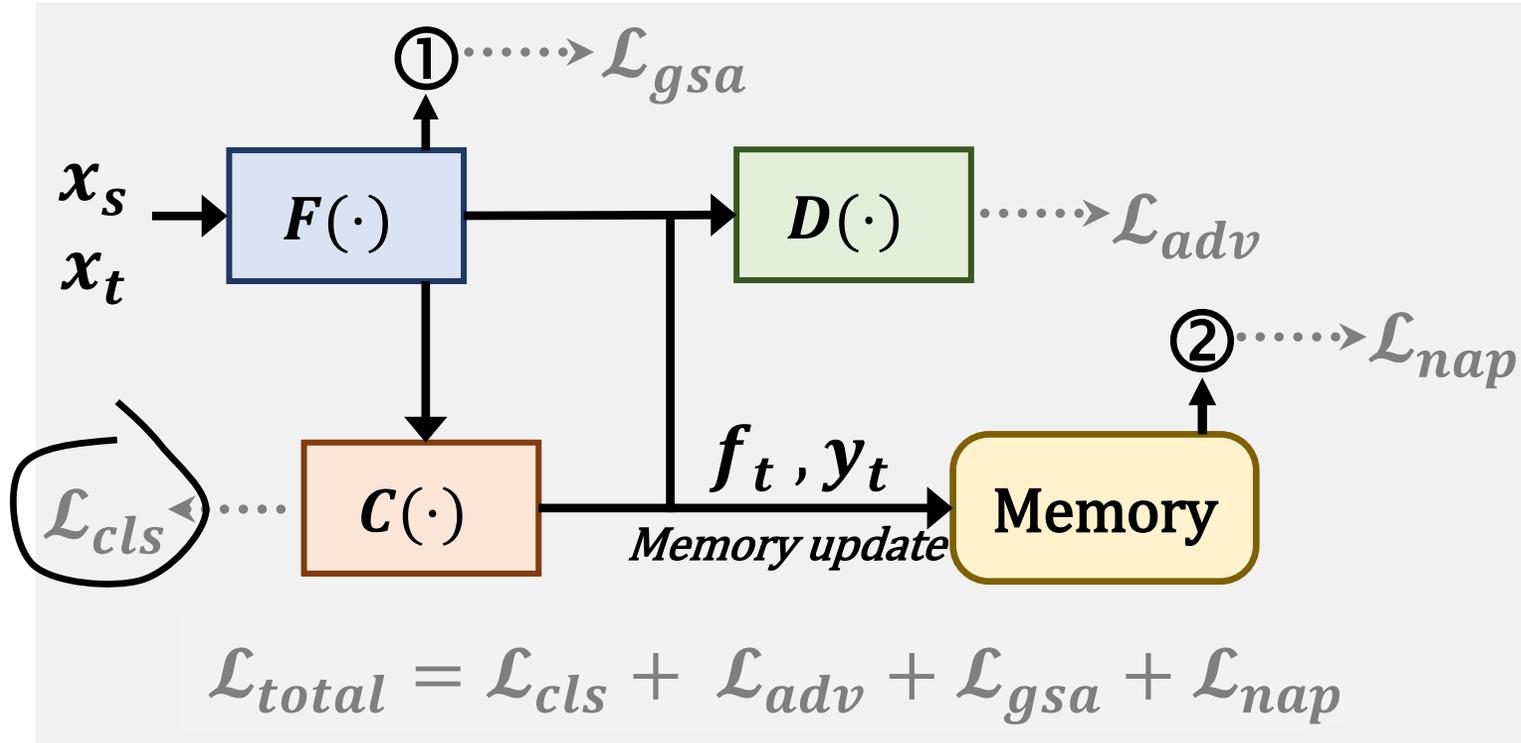
# SPA Framework

- Dynamic graph construction
- Graph spectral alignment
- Neighbor-aware propagation



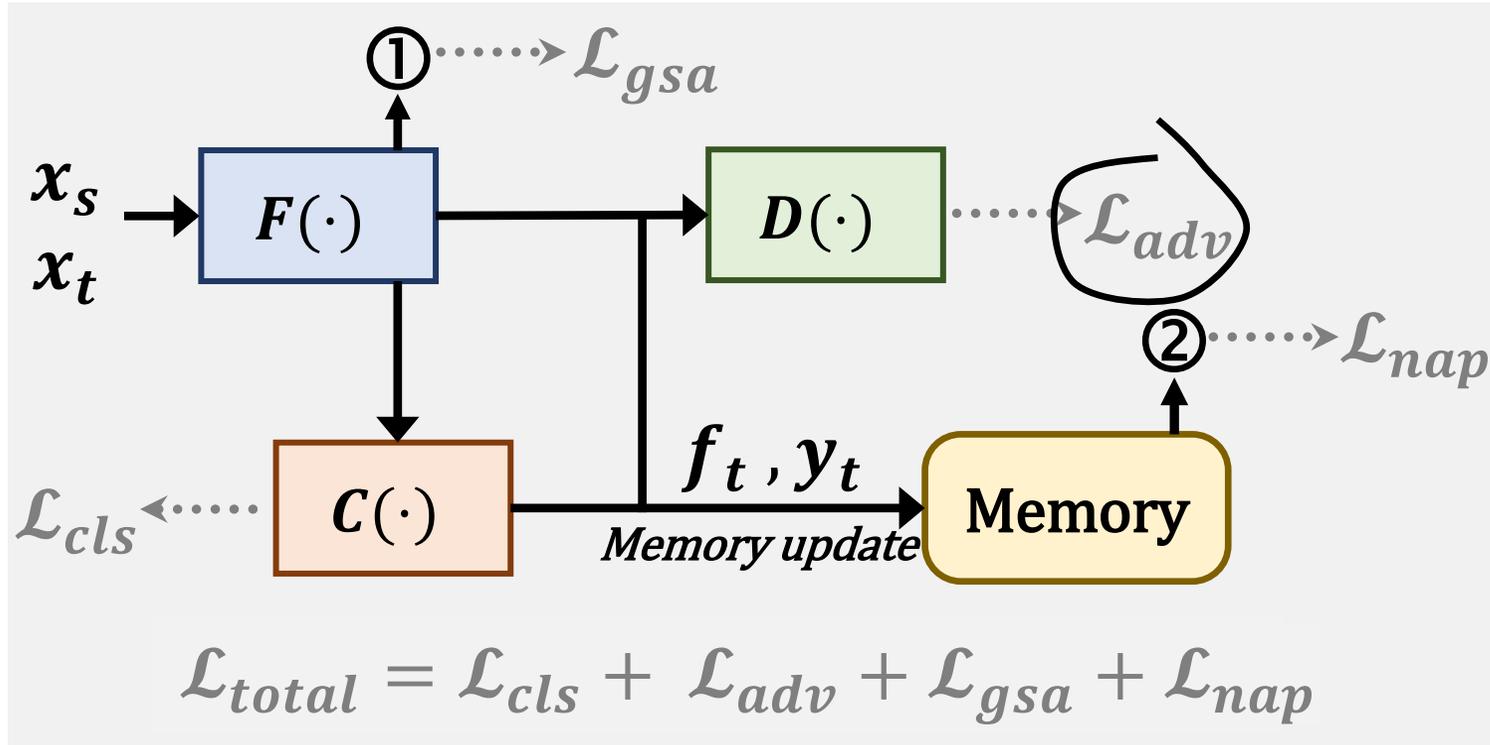
# SPA Framework

$$\mathcal{L}_{cls} = \mathcal{L}_{cross\ entropy}(C(F(x_s)), y_s)$$



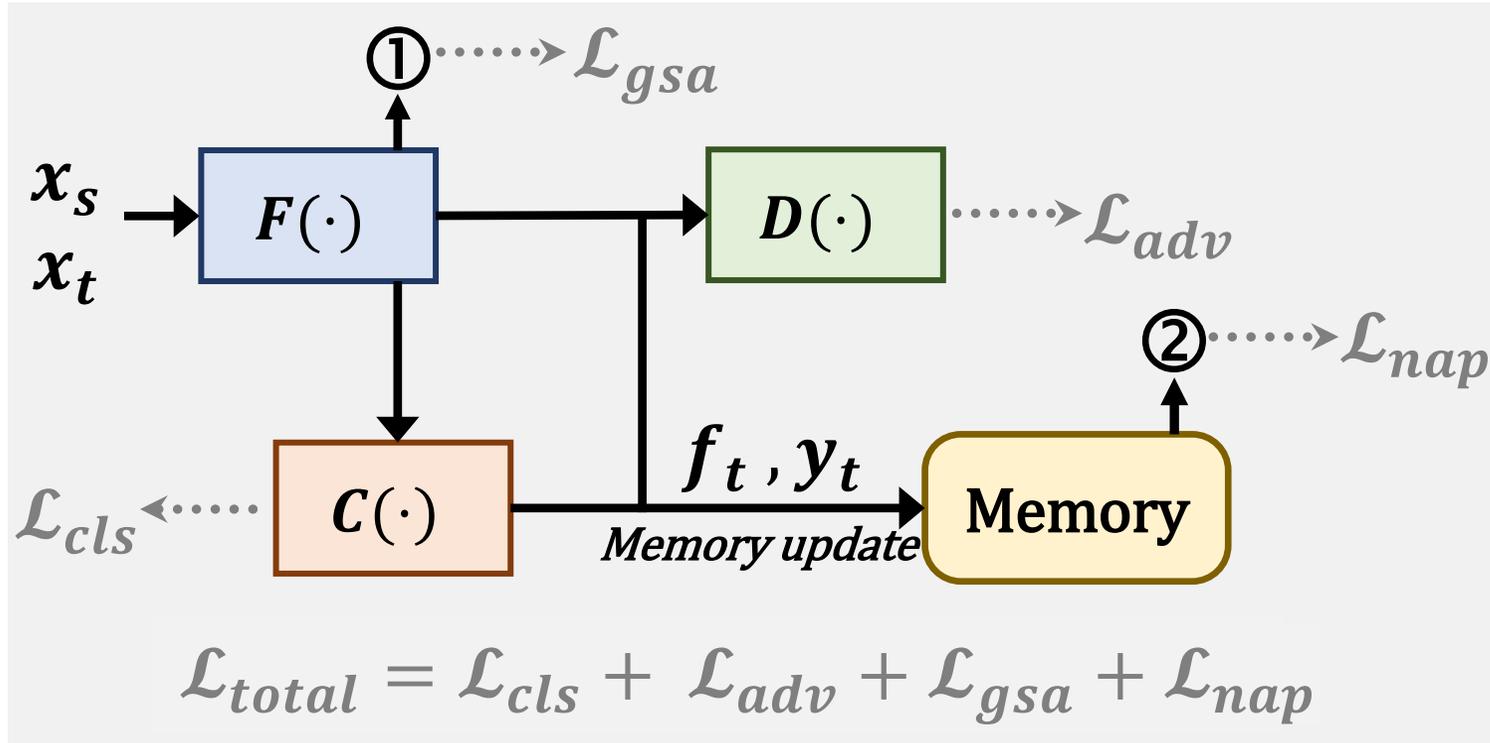
# SPA Framework

$$\mathcal{L}_{adv} = \log[D(F(x_s))] + \log[1 - D(F(x_t))]$$

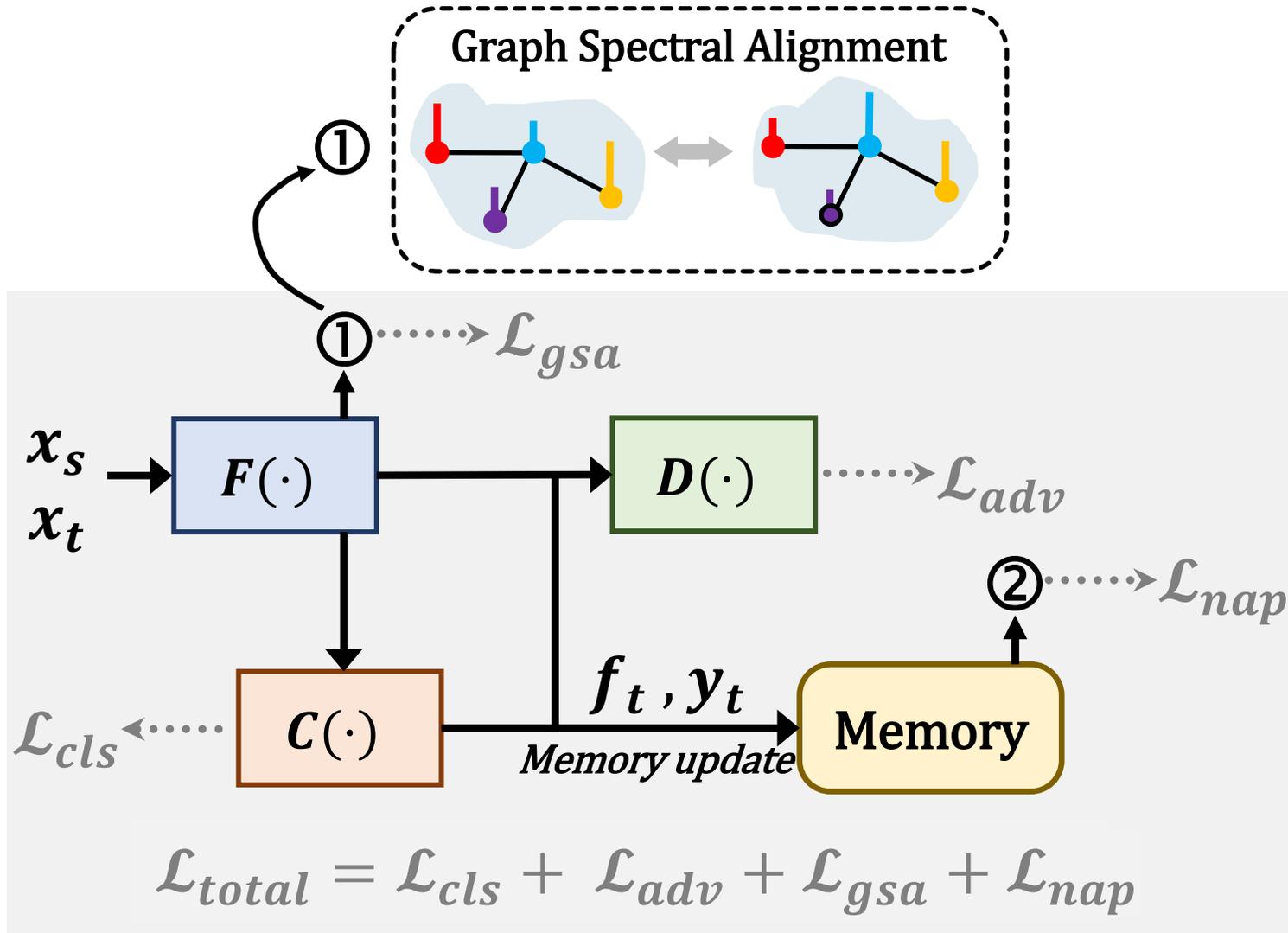


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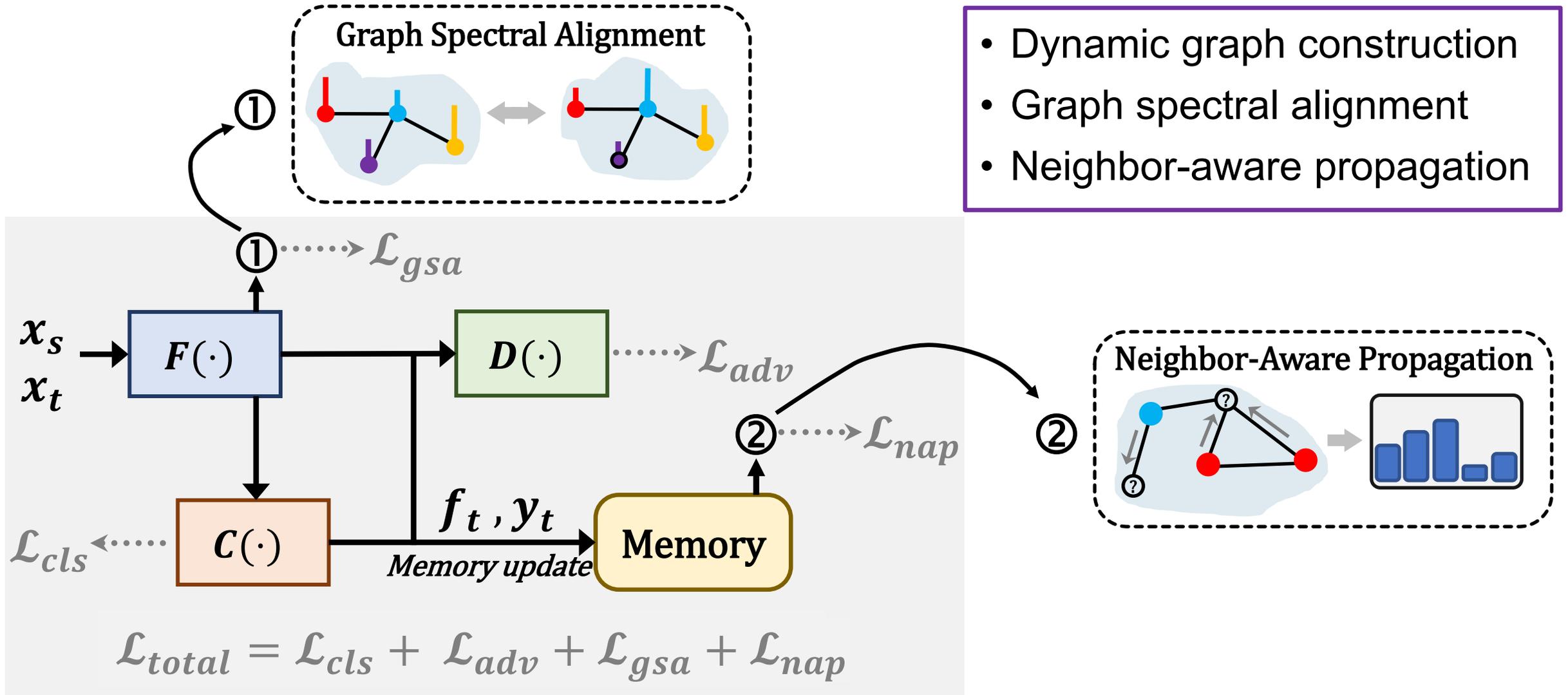


# SPA Framework



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# SPA Framework



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# Dynamic graph construction

- By constructing graphs based on images or text data, both intra-domain relations and inter-domain relations can be exploited.
- Our self-correlation graphs are constructed on source features  $f_s = F(x_s)$  and target features  $f_t = F(x_t)$  respectively.

⇒ Source domain graph  $\mathcal{G}_s = (\mathcal{V}_s, \mathcal{E}_s)$

⇒ Target domain graph  $\mathcal{G}_t = (\mathcal{V}_t, \mathcal{E}_t)$

? inter-domain information

? intra-domain information

# Graph spectral alignment

- To align the distributions of the source and target domains, we prefer implicit graph alignment, avoiding multiple stages of explicit graph matching.
- Inspired by graph Laplacian filters, we propose the definition of spectral distances, projecting domain graphs into eigenspaces and aligning these graphs based on their eigenvalues.

**Definition 1.** (GRAPH LAPLACIANS [77]). *Let  $\mathcal{G} = (\mathcal{V}, \mathcal{E})$  be a finite graph with vertices  $\mathcal{V}$  and weighted edges  $\mathcal{E}$ . Let  $\phi : \mathcal{V} \rightarrow \mathcal{R}$  be a function of the vertices taking values in a ring and  $\gamma : \mathcal{E} \rightarrow \mathcal{R}$  be a weighting function of weighed edges. Then, the graph Laplacian  $\Delta$  acting on  $\phi$  and  $\gamma$  is defined by*

$$(\Delta_{\gamma}\phi)(v) = \sum_{w:d(w,v)=1} \gamma_{wv}[\phi(v) - \phi(w)]$$

*where  $d(w, v)$  is the graph distance between vertices  $w$  and  $v$ , and  $\gamma_{wv}$  is the weight value on the edge  $wv \in \mathcal{E}$ .*

# Graph spectral alignment

**Definition 2.** (SPECTRAL DISTANCES) Given two simple and nonisomorphic graphs  $\mathcal{G}_s$  and  $\mathcal{G}_t$  on  $n$  vertices with the spectra of Laplacians  $\Lambda_s = \{\lambda_i^s\}_{i=1}^n$  with  $\lambda_1^s \geq \lambda_2^s \geq \dots \geq \lambda_n^s$  and  $\Lambda_t = \{\lambda_i^t\}_{i=1}^n$  with  $\lambda_1^t \geq \lambda_2^t \geq \dots \geq \lambda_n^t$  respectively. Define the spectral distance between  $\mathcal{G}_s$  and  $\mathcal{G}_t$  as

$$\sigma(\mathcal{G}_s, \mathcal{G}_t) = \|\Lambda_s - \Lambda_t\|_p, \quad p \geq 1$$

- The graph spectral alignment penalty is defined as  $\mathcal{L}_{gsa} = \sigma(\mathcal{G}_s, \mathcal{G}_t)$ , which measures the discrepancy of two graphs on spectrum space.
- Minimizing this penalty decreases the distance of source domain graphs and target domain graphs.

✓ inter-domain information  
? intra-domain information

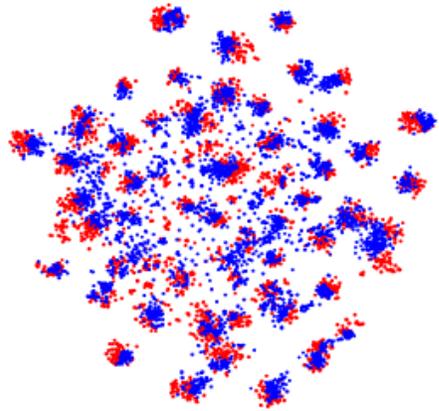
# Neighbor-aware propagation

- The well-trained source domain naturally forms tight clusters in the latent space. After aligning via the aforementioned graph spectral penalty, the rich topological information is coarsely transferred to the target domain.
- We perform further **fine-grained intra-domain alignment** by encouraging message propagation within the target domain graph.
- The normalized probability  $\hat{q}_{i,c}$  is yielded from voted probability  $q_{i,c} = \sum p_{j,c}^m$ , where  $p_i^m$  denotes sample probability stored in the memory bank

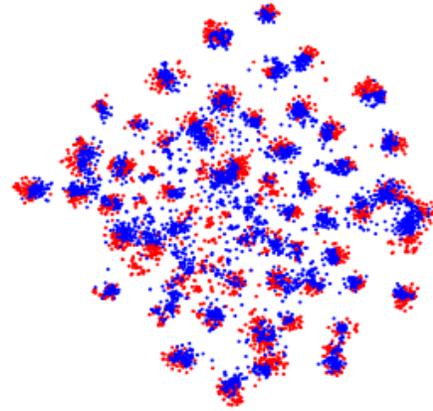
$$\mathcal{L}_{nap} = -\alpha \cdot \frac{1}{N_t} \sum_{i=1} \hat{q}_{i,\hat{y}_i} \log p_{i,\hat{y}_i}$$

- ✓ inter-domain information
- ✓ intra-domain information

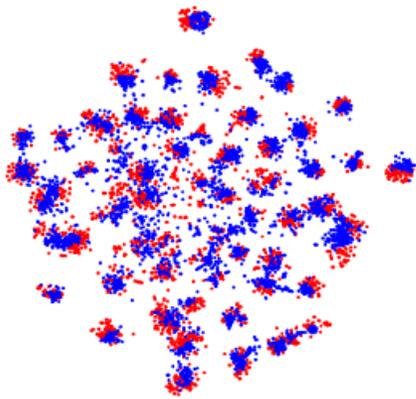
# Transferability vs. Discriminability



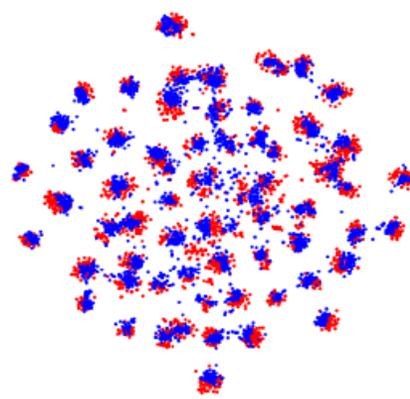
(a) DANN



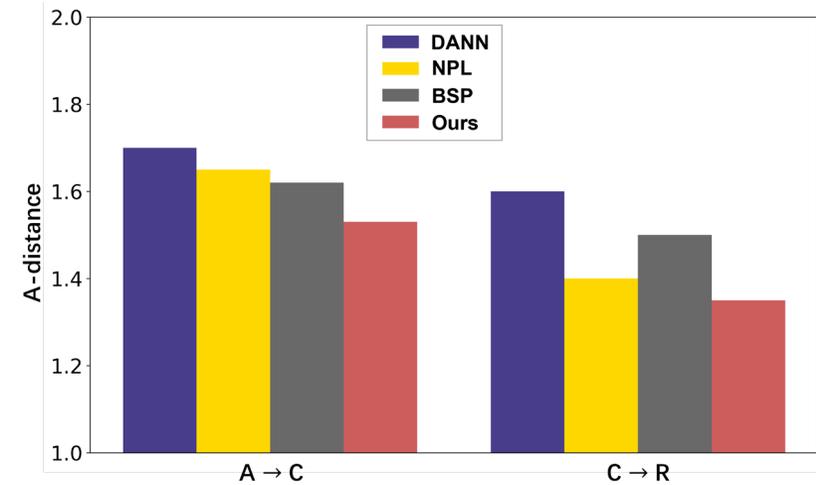
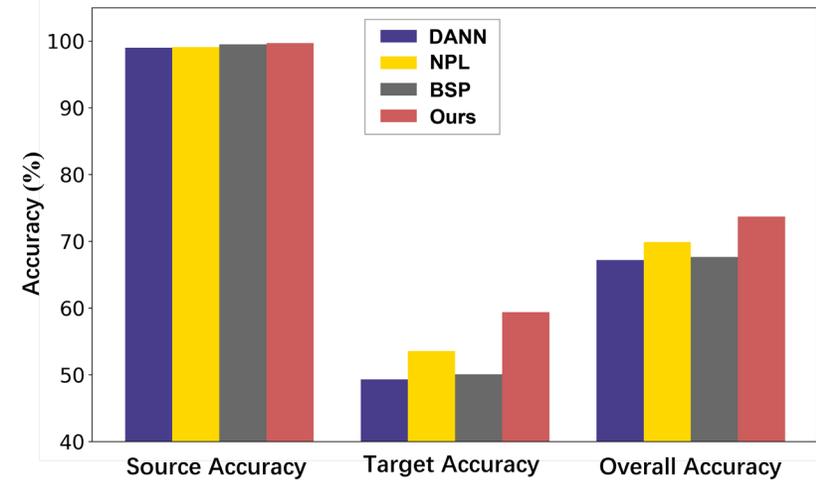
(b) BSP



(c) NPL



(d) SPA



# Experiments

Method	C→P	C→R	C→S	P→C	P→R	P→S	R→C	R→P	R→S	S→C	S→P	S→R	Avg.
<i>Source Only</i> [24]	32.7	50.6	39.4	41.1	56.8	35.0	48.6	48.8	36.1	49.0	34.8	46.1	43.3
DAN [50]	38.8	55.2	43.9	45.9	59.0	40.8	50.8	49.8	38.9	56.1	45.9	55.5	48.4
DANN [17]	37.9	54.3	44.4	41.7	55.6	36.8	50.7	50.8	40.1	55.0	45.0	54.5	47.2
BCDM [44]	38.5	53.2	43.9	42.5	54.5	38.5	51.9	51.2	40.6	53.7	46.0	53.4	47.3
MCD [65]	37.5	52.9	44.0	44.6	54.5	41.6	52.0	51.5	39.7	55.5	44.6	52.0	47.5
ADDA [73]	38.4	54.1	44.1	43.5	56.7	39.2	52.8	51.3	40.9	55.0	45.4	54.5	48.0
CDAN [51]	39.9	55.6	45.9	44.8	57.4	40.7	56.3	52.5	44.2	55.1	43.1	53.2	49.1
MCC [30]	40.1	56.5	44.9	46.9	57.7	41.4	56.0	53.7	40.6	58.2	45.1	55.9	49.7
JAN [52]	40.5	56.7	45.1	47.2	59.9	43.0	54.2	52.6	41.9	56.6	46.2	55.5	50.0
MDD [41]	42.9	59.5	47.5	48.6	59.4	42.6	58.3	53.7	46.2	58.7	46.5	57.7	51.8
SDAT [61]	41.5	57.5	47.2	47.5	58.0	41.8	56.7	53.6	43.9	58.7	48.1	57.1	51.0
Leco [80]	44.1	55.3	48.5	49.4	57.5	45.5	58.8	55.4	46.8	61.3	51.1	57.7	52.6
SPA (Ours)	54.3	70.9	56.1	59.3	71.5	51.8	64.6	59.6	52.1	66.0	57.4	70.6	61.2

- DomainNet for inductive UDA
- SPA consistently outperforms than various of DA methods with the average accuracy of 61.2%, which is 8.6% higher than the recent work Leco, demonstrating the superiority of SPA in this inductive setting.

# Experiments

Method	A→C	A→P	A→R	C→A	C→P	C→R	P→A	P→C	P→R	R→A	R→C	R→P	Avg.
<i>Source Only</i> [24]	34.9	50.0	58.0	37.4	41.9	46.2	38.5	31.2	60.4	53.9	41.2	59.9	46.1
DANN [17]	45.6	59.3	70.1	47.0	58.5	60.9	46.1	43.7	68.5	63.2	51.8	76.8	57.6
CDAN [51]	50.7	70.6	76.0	57.6	70.0	70.0	57.4	50.9	77.3	70.9	56.7	81.6	65.8
BSP [9]	52.0	68.6	76.1	58.0	70.3	70.2	58.6	50.2	77.6	72.2	59.3	81.9	66.3
NPL [38]	54.1	74.1	78.4	63.3	72.8	74.0	61.7	51.0	78.9	71.9	56.6	81.9	68.2
GVB [13]	57.0	74.7	79.8	64.6	74.1	74.6	65.2	55.1	81.0	74.6	59.7	84.3	70.4
MCC [30]	56.3	77.3	80.3	67.0	77.1	77.0	66.2	55.1	81.2	73.5	57.4	84.1	71.0
BNM [12]	56.7	77.5	81.0	67.3	76.3	77.1	65.3	55.1	82.0	73.6	57.0	84.3	71.1
MetaAlign [82]	59.3	76.0	80.2	65.7	74.7	75.1	65.7	56.5	81.6	74.1	61.1	85.2	71.3
ATDOC [47]	58.3	78.8	82.3	69.4	78.2	78.2	67.1	56.0	82.7	72.0	58.2	85.5	72.2
FixBi [55]	58.1	77.3	80.4	67.7	79.5	78.1	65.8	57.9	81.7	76.4	62.9	86.7	72.7
SDAT [61]	58.2	77.1	82.2	66.3	77.6	76.8	63.3	57.0	82.2	74.9	64.7	86.0	72.2
NWD [7]	58.1	79.6	83.7	67.7	77.9	78.7	66.8	56.0	81.9	73.9	60.9	86.1	72.6
SPA (Ours)	60.4	79.7	84.5	73.6	81.3	82.1	72.2	58.0	85.2	77.4	61.0	88.1	75.3

- OfficeHome for UDA
- This table shows the average accuracy of SPA is 75.3%, achieving the best accuracy, which is 2.6% higher than the second highest method FixBi and 2.7% higher than the recent work NWD

# SPA: A Graph Spectral Alignment Perspective for Domain Adaptation

- ✓ Propose a novel **graph spectral alignment** perspective for DA
- ✓ Balance **inter-domain transferability** and **intra-domain discriminability**
- ✓ Achieve **SOTA** performance in unsupervised DA and semi-supervised DA

**Thanks!**

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Github: <https://github.com/CrownX/SPA>





**Thanks!**