Advice Querying under Budget Constraint for Online Algorithms

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Algorithms with predictions

- **Online algorithms**
  - Minimization (or maximization) problem
  - data revealed sequentially $X = (x_1, \dots, x_k)$
  - Competitive ratio: $\mathrm{CR}(\mathrm{ALG}) = \sup_{\mathcal{I}} \frac{\mathrm{ALG}(X)}{\mathrm{OPT}(X)}$

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- **Learning-augmented algorithms**
  - predictions $Y = (y_1, \ldots, y_k)$ of $X = (x_1, \ldots, x_k)$, $\eta = d(X, Y)$
  - **Robustness**: $\eta \to \infty \implies \text{CR}(\text{ALG}, Y) = O(\text{CR}(\text{ALG}))$
  - **Consistency**: $\eta \to 0 \implies \text{CR}(\text{ALG}, Y) = O(1)$
  - λ: parameter for tuning the consistency/robustness tradeoff

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  - one prediction for each unknown parameter
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- **What if ...**
  - limited number of predictions?
  - the algorithm chooses when to query them?
  - guarantees on their quality?

Related Work

Aditya Bhaskara, Ashok Cutkosky, Ravi Kumar, and Manish Purohit.
Logarithmic regret from sublinear hints.
Advances in Neural Information Processing Systems, 34:28222–28232, 2021.

Sungjin Im, Ravi Kumar, Aditya Petety, and Manish Purohit.
Parsimonious learning-augmented caching.
In *International Conference on Machine Learning*, pages 9588–9601. PMLR, 2022.

Anupam Gupta, Debmalya Panigrahi, Bernardo Subercaseaux, and Kevin Sun.
Augmenting online algorithms with ε-accurate predictions.
Advances in Neural Information Processing Systems, 35:2115–2127, 2022.

Time-dependent guarantee on the prediction

Ski-rental problem

- cost of renting $= 1$, cost of buying $= b > 1$
- $x =$ unknown number of snow days
- $\text{OPT}(x) = \min(x, b)$, $\text{CR}(\text{DET}) = 2 - \frac{1}{b}$,
  $\text{CR}(\text{RAND}) = \left(1 - (1 - 1/b)^b\right)^{-1} \to \frac{e}{e-1}$

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- **Prediction**
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- **Algorithm**
  - ALG_t Rent until t then run $\mathrm{ALG}(p_t)$
  - $\mathrm{CR}(\mathrm{ALG}_t) \leq \frac{t}{b} + \mathrm{CR}(\mathrm{ALG}(p_t))$

Secretary problem with recommendations

Secretary problem

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- objective: Select the maximum
- $\text{ALG}^* = 1/e$-rule, $\Pr(\text{ALG}^* \text{ succeeds}) = \frac{1}{e}$

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- **Algorithm**
  - adaptive threshold: restarted $1/e$-rule
  - trust the recommendation if $p > p^*$
  - $\Pr(\text{ALG succeeds}) \to 1$ for $B \to \infty$

B-clairvoyant scheduling

Preemptive non-clairvoyant scheduling problem

- N jobs of unknown sizes to be scheduled on a single machine
- objective: minimize the sum of the completion times
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- **Algorithm**
  - query the sizes of B random jobs
  - Run concurrently OPT on the B jobs with known sizes and RR on the $N - B$ jobs with unknown sizes
  - Competitive ratio $2 - \frac{B(B-1)}{N(N-1)}$