



# Responsible AI (RAI) Games and Ensembles

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Code: <https://github.com/yashgupta-7/rai-games>



# Responsible AI (RAI): Introduction and Motivation

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- **Motivation:** AI is increasingly being used in high-stakes decision-making contexts such as hiring, criminal justice, and healthcare.
- **Setting:** Under the umbrella of “responsible AI”, an emerging line of work has attempted to formalize desiderata ranging over ethics, fairness, robustness, and safety, many of which can be written as *min-max problems* involving optimizing some worst-case loss under a set of predefined distributions.
- **Problem:** majority of recent work around these problems is fragmented and usually focuses on optimizing one of these aspects at a time (DRO [Namkoong and Duchi, 2017, Duchi and Namkoong, 2018], GDRO [Sagawa et al., 2019], CVaR [Zhai et al., 2021a], Distribution Shift [Hashimoto et al., 2018, Zhai et al., 2021b]).
- **Proposal:** a general game-theoretic framework for solving these problems and learning responsible AI models. We propose practical algorithms to solve these games, as well as statistical analyses of solutions of these games.



# Problem Setup

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- standard supervised prediction setting: input random variable  $X \in \mathcal{X} \subseteq \mathbb{R}^d$ , output random variable  $Y \in \mathcal{Y}$ , and samples  $S = \{(x_i, y_i)\}_{i=1}^n$  drawn from a distribution  $P_{\text{data}}$  over  $\mathcal{X} \times \mathcal{Y}$
- The empirical distribution  $\hat{P}_{\text{data}}$  over the samples, set  $H$  of hypothesis functions  $h : \mathcal{X} \mapsto \mathcal{Y}$
- Goodness of a predictor via a loss function  $\ell : \mathcal{Y} \times \mathcal{Y} \mapsto \mathbb{R}$ , which yields the empirical risk:  
$$\hat{R}(h) = \mathbb{E}_{\hat{P}_{\text{data}}} \ell(h(x), y) \quad \text{where} \quad \mathbb{E}_{\hat{P}_{\text{data}}} (f(x, y)) = \frac{1}{n} \sum_{i=1}^n f(x_i, y_i).$$
- Apart from having low expected risk,  $h$  is required to have certain properties.  
e.g. robustness, fairness w.r.t subpopulations, superior tail performance, resistance to adversarial attacks, etc - cast all these subproblems into an umbrella term “Responsible AI”.

# RAI Risks - I

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- Do not wish to compute an unweighted average over training samples - due to RAI considerations.

**Definition 1 (RAI Risks)** Given a set of samples  $\{(x_i, y_i)\}_{i=1}^n$ , we define the class of empirical RAI risks (for Responsible AI risks) as:  $\hat{R}_{W_n}(h) = \sup_{w \in W_n} \mathbb{E}_w(h(x), y)$ , where  $W_n \subseteq \Delta_n$ , is some set of sample weights (a.k.a uncertainty set), and  $\mathbb{E}_w(f(x, y)) = \sum_{i=1}^n w_i f(x_i, y_i)$ .

- Given the empirical RAI risk of a hypothesis - naturally wish to obtain the hypothesis that minimizes the empirical RAI risk
- Can be seen as solving a zero-sum game

**Definition 2 (RAI Games)** Given a set of hypothesis  $H$ , and a RAI sample weight set  $W_n$ , the class of RAI games is given as:  $\min_{h \in H} \max_{w \in W_n} \mathbb{E}_w(h(x), y)$ .

# RAI Risks - II



- Various choices of  $\mathbf{W}_n$  give rise to various RAI risks.

Name	$\mathbf{W}_n$	Description
Empirical Risk Minimization	$\{\hat{P}_{\text{data}}\}$	object of focus in most of ML/AI
Worst Case Margin	$\Delta_n$ , entire probability simplex	used for designing margin-boosting algorithms [Warmuth et al., 2006, Bartlett et al., 1998]
Soft Margin	$\{w : KL(w  \hat{P}_{\text{data}}) \leq \rho_n\}$	used in the design of AdaBoost [Freund and Schapire, 1995]
$\alpha$ -Conditional Value at Risk (CVaR)	$\{w : w \in \Delta_n, w \preceq \frac{1}{\alpha n}\}$	used in fairness [Zhai et al., 2021a, Sagawa et al., 2019]
Distributionally Robust Optimization (DRO)	$\{w : D(w  \hat{P}_{\text{data}}) \leq \rho_n\}$	various choices for $D$ have been studied $f$ -divergence [Duchi and Namkoong, 2018]
Group DRO	$\{\hat{P}_{\text{data}}(G_1), \hat{P}_{\text{data}}(G_2), \dots, \hat{P}_{\text{data}}(G_K)\}$ $\hat{P}_{\text{data}}(G_i)$ is dist. of $i^{\text{th}}$ group	used in group fairness, agnostic federated learning [Mohri et al., 2019]

Table 1: Various ML/AI problems that fall under the umbrella of RAI risks.

# RAI Games - Moving to ensembles

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- Good worst-case performance over the sample weight set  $W_n$  is generally harder, especially for a simpler set of hypotheses
- Natural to consider deterministic ensemble models
  - Gives us more powerful classes

**Definition 3 (Deterministic Ensemble)** Consider the problem of classification, where  $\mathcal{Y}$  is a discrete set. Given a hypothesis class  $H$ , a deterministic ensemble is specified by some distribution  $Q \in \Delta_H$ , and is given by:  $h_{det;Q}(x) = \arg \max_{y \in \mathcal{Y}} \mathbb{E}_{h \sim Q} \mathbb{I}[h(x) = y]$ . Correspondingly, we can write the deterministic ensemble RAI risk as  $\hat{R}_{W_n}(h_{det;Q}(x)) = \max_{w \in W_n} \mathbb{E}_w \ell(h_{det;Q}(x), y)$ .

- This admits a class of deterministic RAI games

**Definition 4 (Deterministic Ensemble RAI Games)** Given a set of hypothesis  $H$ , a RAI sample weight set  $W_n$ , the class of RAI games for deterministic ensembles over  $H$  is given as:

$$\min_{Q \in \Delta_H} \max_{w \in W_n} \mathbb{E}_w \ell(h_{det;Q}(x), y).$$

# RAI Games - Moving to *random* ensembles

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- Aforementioned game is computationally less amenable because of the **non-smooth** nature of de-randomized predictions.
- To this end, we consider the following randomized ensembles:

**Definition 5 (Randomized Ensemble)** *Given a hypothesis class  $H$ , a randomized ensemble is specified by some distribution  $Q \in \Delta_H$ , and is given by:  $\mathbb{P}[h_{rand;Q}(x) = y] = \mathbb{E}_{h \sim Q} \mathbb{I}[h(x) = y]$ . Similarly, we can define its corresponding randomized ensemble RAI risk:  $\widehat{R}_{rand;W_n}(Q) = \max_{w \in W_n} \mathbb{E}_{h \sim Q} \mathbb{E}_w \ell(h(x), y)$ .*

**Definition 6 (Randomized Ensemble RAI Games)** *Given a set of hypothesis  $H$ , a RAI sample weight set  $W_n$ , the class of mixed RAI games is given as:*

$$\min_{Q \in \Delta_H} \max_{w \in W_n} \mathbb{E}_{h \sim Q} \mathbb{E}_w \ell(h(x), y). \quad (1)$$

- Much better class of zero-sum games
  - **linear** in both the hypothesis distribution  $P$  well as the sample weights
  - if the sample weight set is convex, is a **convex-concave** game.
  - under some mild conditions, this game has a **Nash** equilibrium

# RAI Algorithms - I

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- **Game Play** - Both players rely on no-regret algorithms to decide their next action
  - Follow-The-Regularized-Leader (FTRL) update for weights
  - Best Response (BR) update for hypotheses

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**Algorithm 1** Game play algorithm for solving Equation (1)

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**Input:** Training data  $\{(x_i, y_i)\}_{i=1}^n$ , loss function  $\ell$ , constraint set  $W_n$ , hypothesis set  $H$ , strongly concave regularizer  $R$  over  $W_n$ , learning rates  $\{\eta^t\}_{t=1}^T$

- 1: **for**  $t \leftarrow 1$  to  $T$  **do**
  - 2:     **FTRL:**  $w^t \leftarrow \operatorname{argmax}_{w \in W_n} \sum_{s=1}^{t-1} \mathbb{E}_w \ell(h^s(x), y) + \eta^{t-1} \operatorname{Reg}(w)$
  - 3:     **BR:**  $h^t \leftarrow \operatorname{argmin}_{h \in H} \mathbb{E}_{w^t} \ell(h(x), y)$
  - 4: **end for**
  - 5: **return**  $P^T = \frac{1}{T} \sum_{t=1}^T w^t, Q^T = \operatorname{Unif}\{h^1, \dots, h^T\}$
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# RAI Algorithms - II

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- **Greedy** - use Frank Wolfe (FW) for the inner maximization problem
  - when it is smooth, updates given by:

$$Q^t \leftarrow (1 - \alpha^t)Q^{t-1} + \alpha^t G, \quad \text{where } G = \underset{Q}{\operatorname{argmin}} \langle Q, \nabla_Q L(Q^{t-1}) \rangle.$$

- when non-smooth, perform Moreau smoothing

$$L_\eta(Q) = \max_{w \in W_n} \mathbb{E}_{h \sim Q} \mathbb{E}_w \ell(h(x), y) + \eta \operatorname{Reg}(w).$$

- a slightly different AdaBoost-like algorithm by relaxing the simplex constraint on  $Q$

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**Algorithm 2** Greedy algorithms for solving Equation (1)

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**Input:** Training data  $\{(x_i, y_i)\}_{i=1}^n$ , loss function  $\ell$ , constraint set  $W_n$ , hypothesis set  $H$ , strongly concave regularizer  $R$  over  $W_n$ , regularization strength  $\eta$ , step sizes  $\{\alpha^t\}_{t=1}^T$

- 1: **for**  $t \leftarrow 1$  to  $T$  **do**
  - 2:      $G^t = \operatorname{argmin}_Q \langle Q, \nabla_Q L_\eta(Q^{t-1}) \rangle$
  - 3:     **FW:**  $Q^t \leftarrow (1 - \alpha^t)Q^{t-1} + \alpha^t G^t$  / **Gen-AdaBoost:**  $Q^t \leftarrow Q^{t-1} + \alpha^t G^t$
  - 4: **end for**
  - 5: **return**  $Q^T$
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# Experiments

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- **Goal:** demonstrate the generality of proposed RAI methods by studying a well studied problem i.e. **subpopulation shift** under various settings
  - Domain-oblivious (DO): we do not know the sub-populations [[Hashimoto et al., 2018](#), [Lahoti et al., 2020](#)]
    - $\chi^2$ -DRO constraint set to control
  - Domain-aware (DA): where we know the sub-populations [[Sagawa et al., 2019](#)]
    - Group DRO constraint set
  - Partially domain-aware (PDA): where only some might be known
    - intersection over Group DRO constraints over the known domains and  $\chi^2$  constraints to control
- **Baselines -**
  - Deterministic classifiers trained on empirical risk (ERM) and DRO risks
    - the quasi-online algorithm for Group DRO [[Sagawa et al., 2019](#)] (Online GDRO)
    - ITLM-inspired SGD algorithm [[Zhai et al., 2021b](#), [Shen and Sanghavi, 2018](#)] for  $\chi^2$  DRO (SGD ( $\chi^2$ ))
  - Ensemble models AdaBoost [[Schapire, 1999](#)].

# Experiments - Results



- RAI-FW and RAI-GA methods significantly improve the worst-case performance with only 3-5 base learners across all datasets in all three settings, while maintaining average case performance.
- The plug-and-play framework allows for several different to enhance various responsible AI qualities at once. RAI is able to optimize effectively for both known and unknown subpopulations

Table 2: (Table 1 in the paper) Mean and worst-case expected loss for baselines, RAI-GA and RAI-FW. (Complex) indicates the use of larger models. Constraint sets  $W_n$  are indicated in (.). Each experiment is carried out over three random seeds and confidence intervals are reported.

Setting	Algorithm	COMPAS		CIFAR-10 (Imbalanced)		CIFAR10		CIFAR100	
		Average	Worst Group	Average	Worst Class	Average	Worst Class	Average	Worst Class
DO (Complex)	ERM	31.3 ±0.2	31.7 ±0.1	12.1 ±0.3	30.4 ±0.2	8.3 ±0.2	21.3 ±0.5	25.2 ±0.2	64.0 ±0.7
	RAI-GA ( $\chi^2$ )	31.3 ±0.2	<b>31.2</b> ±0.2	11.7 ±0.4	<b>29.0</b> ±0.3	8.2 ±0.1	19.0 ±0.1	25.6 ±0.4	<b>56.8</b> ±0.8
	RAI-FW ( $\chi^2$ )	31.2 ±0.1	31.4 ±0.3	11.9 ±0.1	29.1 ±0.2	8.0 ±0.3	<b>15.4</b> ±0.4	25.4 ±0.2	58.0 ±1.1
DO	ERM	32.1 ±0.3	34.6 ±0.4	14.2 ±0.1	33.6 ±0.3	11.4 ±0.4	27.0 ±0.1	27.1 ±0.3	66.0 ±1.1
	AdaBoost	31.8 ±0.4	32.6 ±0.3	15.2 ±0.2	40.6 ±0.2	12.0 ±0.1	28.7 ±0.3	28.1 ±0.2	72.2 ±1.2
	SGD ( $\chi^2$ )	32.0 ±0.2	33.7 ±0.2	13.3 ±0.3	31.7 ±0.4	11.3 ±0.3	24.7 ±0.1	27.4 ±0.1	65.9 ±1.2
	RAI-GA ( $\chi^2$ )	31.5 ±0.2	33.2 ±0.3	14.0 ±0.1	32.2 ±0.2	10.8 ±0.4	25.0 ±0.2	27.4 ±0.4	65.0 ±0.8
	RAI-FW ( $\chi^2$ )	31.6 ±0.1	<b>32.5</b> ±0.5	13.9 ±0.1	32.6 ±0.3	10.9 ±0.4	<b>23.4</b> ±0.2	27.5 ±0.1	<b>63.8</b> ±0.6
DA	Online GDRO	31.7 ±0.2	32.2 ±0.3	13.1 ±0.2	26.6 ±0.2	11.2 ±0.1	21.7 ±0.3	27.3 ±0.1	57.0 ±0.5
	RAI-GA (Group)	32.0 ±0.1	32.7 ±0.1	13.0 ±0.3	27.3 ±0.4	11.5 ±0.1	22.4 ±0.2	27.4 ±0.2	56.6 ±1.1
	RAI-FW (Group)	32.1 ±0.2	32.3 ±0.2	13.0 ±0.2	<b>26.0</b> ±0.1	11.4 ±0.3	<b>20.3</b> ±0.1	27.9 ±0.2	<b>52.9</b> ±0.9
PDA	Online GDRO	31.5 ±0.1	32.7 ±0.2	13.4 ±0.1	32.2 ±0.2	11.3 ±0.2	25.2 ±0.1	27.7 ±0.2	64.0 ±0.8
	RAI-GA (Group $\cap \chi^2$ )	31.4 ±0.4	32.9 ±0.2	13.0 ±0.3	30.1 ±0.1	10.8 ±0.2	<b>23.7</b> ±0.2	27.5 ±0.1	62.5 ±0.6
	RAI-FW (Group $\cap \chi^2$ )	31.8 ±0.2	<b>32.3</b> ±0.1	13.5 ±0.3	<b>29.4</b> ±0.3	11.2 ±0.4	24.0 ±0.2	27.9 ±0.3	<b>58.9</b> ±0.7



**Thank You!**