Online Pricing for Multi-User Multi-Item Markets

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Introduction

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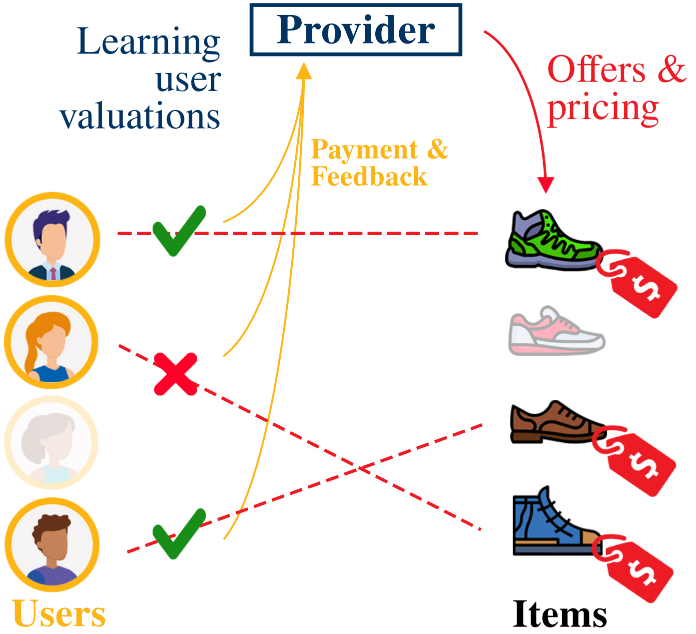
- **Learning**
- **Accept/reject**
- **Goal:**
- *offers* *pricing*



Problem Setting

- □□□□□□□□□□□$\mathcal{N}$□□□□□□□□□N□
- □□□□□□□□ □□$\mathcal{I}$□□□□□□□□□M□

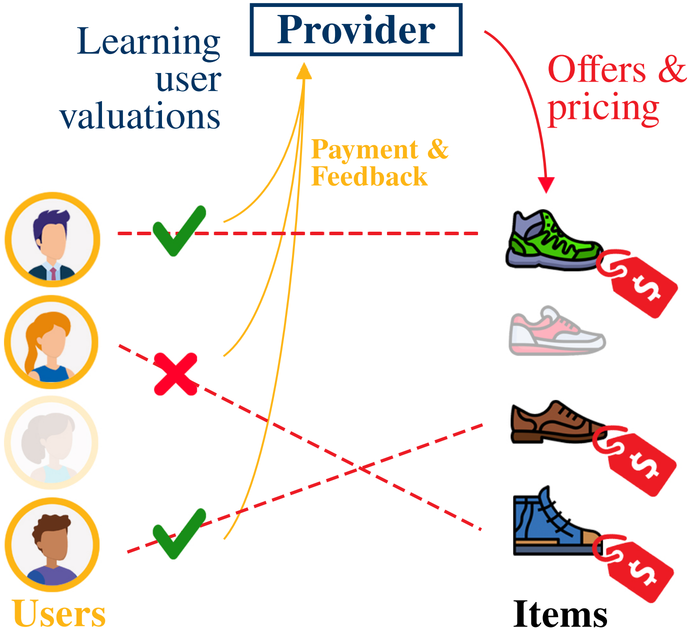
At each round $t \in 1, \ldots, T$:

- A subset of users $\mathcal{D}^t \subseteq \mathcal{N}$ are active.
- A subset of items $\mathcal{E}^t \subseteq \mathcal{I}$ are available for sale.
- Each user u has a different valuation v_{ui}^t for each item i.
- The provider decides on

$$\text{offers } \mathbf{X}^t \in \{0,1\}^{N \times M} \text{ and prices } \mathbf{p}^t \in \mathbb{R}_+^M.$$

- User u accepts an offered item i if and only if

$$v_{ui}^t \geq p_i^t.$$



Goal

- [illegible]

$$\mathcal{R}(T, \boldsymbol{\pi}) = \mathrm{OPT} - \sum_{t=1}^{T} \sum_{u \in \mathcal{D}^t} \sum_{i \in \mathcal{E}^t} x_{ui}^t \, p_i^t \, \mathbb{1}\{v_{ui}^t \geq p_i^t\}$$

- OPT *denotes the revenue of the optimal algorithm (separately for each different valuation model).*

Main Theoretical Results

Main Theoretical Results

- **Our algorithms are optimal up to logarithmic factors!**

Model	Regret Upper Bound	Regret Lower Bound
Fixed Valuations	$O\left(NM \log\log(LT)\right)$	$\Omega\left(NM \log\log(LT/NM)\right)$
Random Experiences	$\tilde{O}\left(\sqrt{NMLT}\right)$	$\Omega\left(\sqrt{NMLT}\right)$
Random Valuations	$\tilde{O}\left(\sqrt{NMLT}\right)$	$\Omega\left(\sqrt{NMLT}\right)$

- **Load parameter:** $L = \max_{1 \le t \le T} \left\{ \min\left(|\mathcal{D}^t|, |\mathcal{E}^t| \right) \right\}$

Model 1: Fixed Valuations

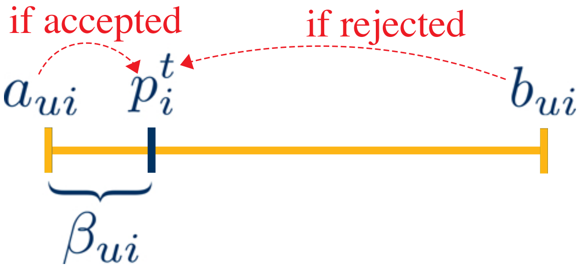
- **do not change over time**
- v_{ui}

$$v_{ui}^t = v_{ui}$$

$t \in [T]$

Model 1: Fixed Valuations

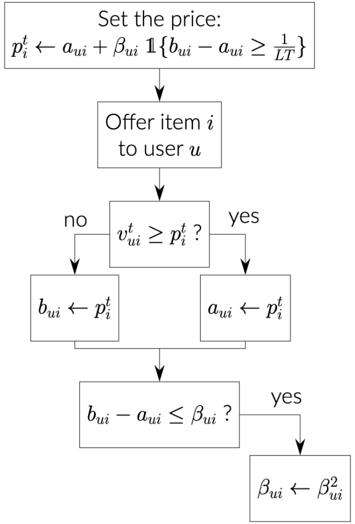
- **Algorithm**



Initialize the interval $[a_{ui}, b_{ui}] = [0, 1]$ for each user-item pair.
Initialize the search step $\beta_{ui} = 1/2$ for each user-item pair.

At each round t:

1. Choose $\mathbf{X}^t$ as the optimum offers under valuations b_{ui}.
2. For each offer (u, i) in $\mathbf{X}^t$ do:



Model 2: Random Experiences

- **average historical experience.**
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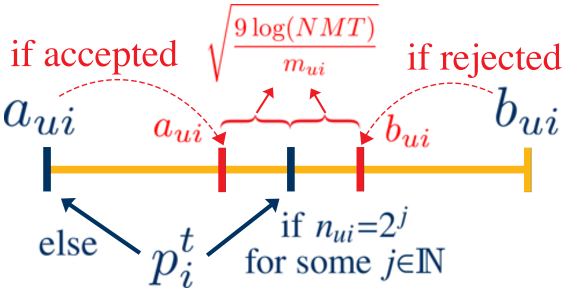
$$v_{ui}^{t} = \text{avg}\{z_{ui}^{\tau} | \tau < t, x_{ui}^{\tau} = 1, v_{ui}^{\tau} \geq p_i^{\tau}\}$$

$t \in [T]$

- z_{ui}^{t}

Model 2: Random Experiences

- **Algorithm**



Initialize the interval $[a_{ui}, b_{ui}] = [0, 1]$ for each user-item pair.

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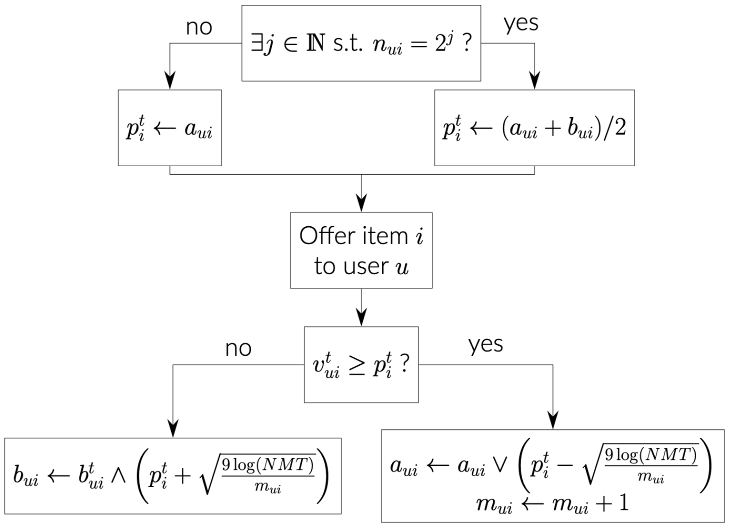
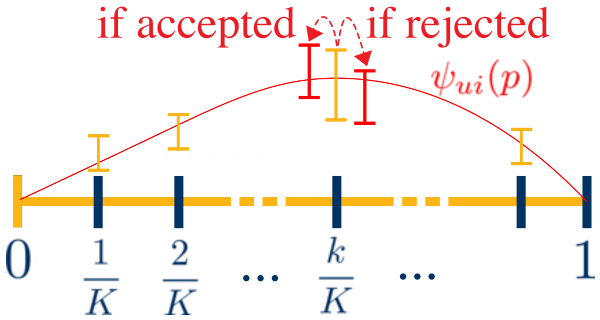


Model 3: Random Valuations

- [illegible] **independently drawn.**
- [illegible] v_{ui}^t [illegible] F_{ui} [illegible]

Model 3: Random Valuations

- **Algorithm**



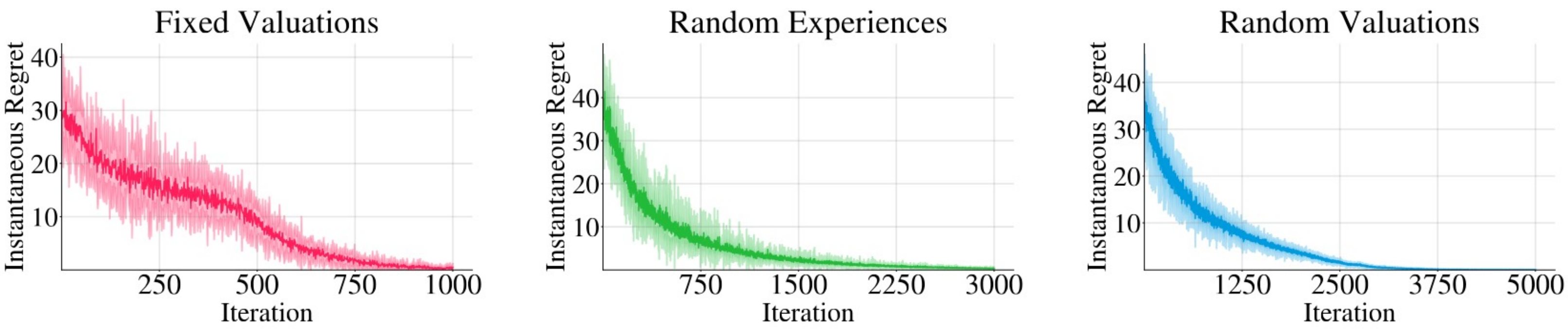
Set $K = (LT/(NM\log(LT)))^{1/4}$.
Quantize prices into $\{1/K, 2/K, \ldots, 1\}$.

At each round t:

1. Set $b_{uik} = \psi_{uik} + \sqrt{\frac{8\log(NMKT)}{n_{uik}}}$
2. Find $b_{ui} = \max_k b_{uik}$.
3. Choose $\mathbf{X}^t$ as the optimum offers under valuations b_{ui}.
4. For each offer (u, i) in $\mathbf{X}^t$ do:
   - Set $k^* = \arg\max_k b_{uik}$.
   - Offer item i to user u at price k^*/K.
   - Update $\psi_{uik^*} \leftarrow \frac{n_{uik^*}\psi_{uik^*} + p_{ui}\mathbb{1}\{v_{ui}^t \geq p_{ui}\}}{n_{uik^*}+1}$.

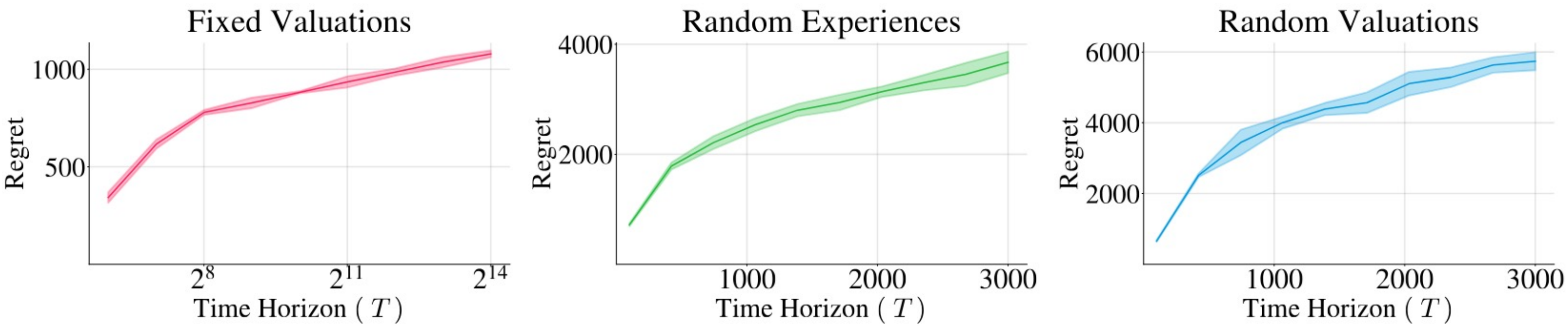
Numerical Experiments

- **Instantaneous Regret** under different valuation models.



Numerical Experiments

- **Regret vs. Time Horizon** under different valuation models.



- Numerical results verify that our algorithms can achieve
  - sub-logarithmic regret under the fixed valuations model,
  - sub-linear regret under random experiences model,
  - sub-linear regret under random valuations model.