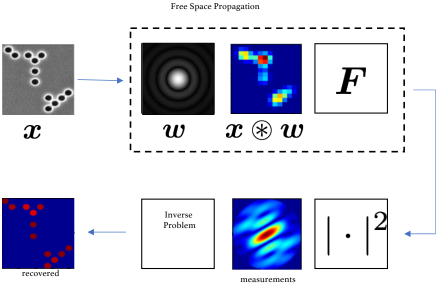
Blind Deconvolutional Phase Retrieval via Convex Programming

Ali Ahmed, Alireza Aghasi, Paul Hand

Funding provided in part by the National Science Foundation

December 5, 2018

Motivation: Blind Deconvolutional Phase Retrieval



Observe: $\hat{\boldsymbol{y}} = |\boldsymbol{F}(\boldsymbol{w} \circledast \boldsymbol{x})|^2$

Find: $\boldsymbol{x} \in \mathbb{R}^L, \boldsymbol{w} \in \mathbb{R}^L$

Assumption: $\boldsymbol{w} = \boldsymbol{B}\boldsymbol{h}, \ \boldsymbol{x} = \boldsymbol{C}\boldsymbol{m},$

$\boldsymbol{B} \in \mathbb{R}^{L \times K}, \ \boldsymbol{C} \in \mathbb{R}^{L \times N}$

Blind Deconvolutional Phase Retrieval (BDPR): Lifting

Observe: $\hat{y}[\ell] = |\boldsymbol{b}_\ell^* \boldsymbol{h}|^2 \cdot |\boldsymbol{c}_\ell^* \boldsymbol{m}|^2$

$\boldsymbol{b}_\ell^*$ is ℓth row of $\boldsymbol{FB}$

$\boldsymbol{c}_\ell^*$ is ℓth row of $\boldsymbol{FC}$

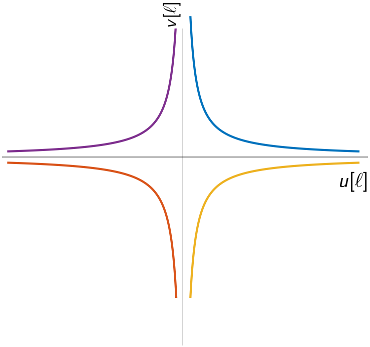
Find: $\boldsymbol{h} \in \mathbb{R}^K, \boldsymbol{m} \in \mathbb{R}^N$

Solve:

$$\begin{aligned} &\underset{\boldsymbol{h},\boldsymbol{m}}{\text{minimize}} \ \|\boldsymbol{h}\|^2 + \|\boldsymbol{m}\|^2 \\ &\text{subject to } \langle \boldsymbol{b}_\ell \boldsymbol{b}_\ell^*, \boldsymbol{X}_1 \rangle \langle \boldsymbol{c}_\ell \boldsymbol{c}_\ell^*, \boldsymbol{X}_2 \rangle = \hat{y}[\ell] \\ &\qquad \boldsymbol{X}_1 = \boldsymbol{h}\boldsymbol{h}^*, \boldsymbol{X}_2 = \boldsymbol{m}\boldsymbol{m}^* \end{aligned}$$

Novel Convex Relaxation via BranchHull

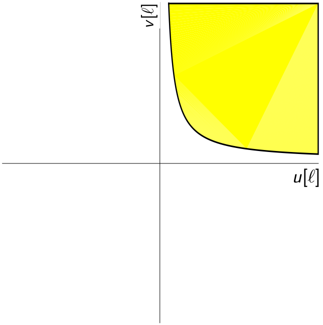
$$\underset{\boldsymbol{X}_1,\boldsymbol{X}_2}{\text{minimize}} \ \ \text{trace}(\boldsymbol{X}_1) + \text{trace}(\boldsymbol{X}_2)$$
$$\text{subject to } \langle \boldsymbol{b}_\ell \boldsymbol{b}_\ell^*, \boldsymbol{X}_1 \rangle \langle \boldsymbol{c}_\ell \boldsymbol{c}_\ell^*, \boldsymbol{X}_2 \rangle = \hat{y}[\ell]$$
$$\boldsymbol{X}_1 \succcurlyeq \boldsymbol{0}, \boldsymbol{X}_2 \succcurlyeq \boldsymbol{0}$$



Hyperbolic constraint set

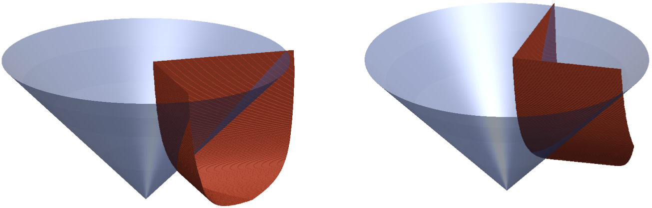
Novel Convex Relaxation via BranchHull

$$\begin{aligned} \underset{\boldsymbol{X}_1, \boldsymbol{X}_2}{\text{minimize}} \quad & \operatorname{trace}(\boldsymbol{X}_1) + \operatorname{trace}(\boldsymbol{X}_2) \\ \text{subject to} \quad & \langle \boldsymbol{b}_\ell \boldsymbol{b}_\ell^*, \boldsymbol{X}_1 \rangle \langle \boldsymbol{c}_\ell \boldsymbol{c}_\ell^*, \boldsymbol{X}_2 \rangle \geq \hat{y}[\ell] \\ & \boldsymbol{X}_1 \succcurlyeq \mathbf{0}, \boldsymbol{X}_2 \succcurlyeq \mathbf{0} \end{aligned}$$



Hyperbolic constraint set

Cartoon of the BranchHull Geometry



Blue: PSD Cone, Red: Boundary of Hyperbolic Constraint

Point in intersection with smallest trace lives along the ridge where hyperbolic constraints are satisfied with equalities.

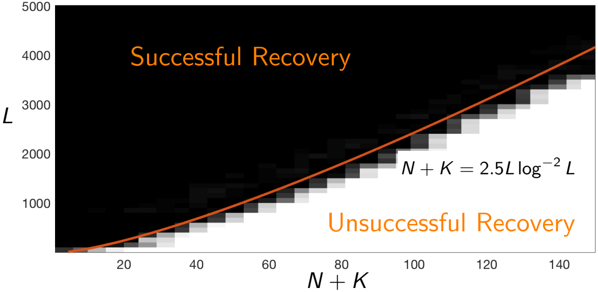
Main Result: Exact Recovery

- Convex program for Blind Deconvolutional Phase Retrieval

$$\begin{aligned} \underset{\boldsymbol{X}_1, \boldsymbol{X}_2}{\text{minimize}} \ & \text{trace}(\boldsymbol{X}_1) + \text{trace}(\boldsymbol{X}_2) \\ \text{subject to} \ & \langle \boldsymbol{b}_\ell \boldsymbol{b}_\ell^*, \boldsymbol{X}_1 \rangle \langle \boldsymbol{c}_\ell \boldsymbol{c}_\ell^*, \boldsymbol{X}_2 \rangle \geq \hat{y}[\ell] \\ & \boldsymbol{X}_1 \succcurlyeq \boldsymbol{0}, \boldsymbol{X}_2 \succcurlyeq \boldsymbol{0}. \end{aligned}$$

- **Theorem** [Ahmed, Aghasi, Hand]: Choose $\boldsymbol{B}$ and $\boldsymbol{C}$ to have i.i.d. standard normal entries. Then, $\boldsymbol{h} \in \mathbb{R}^K$ and $\boldsymbol{m} \in \mathbb{R}^N$ can be exactly recovered (up to global rescaling) with high probability if $L \gtrsim (K + N) \log^2 L$.

Phase Portrait for an ADMM Implementation



Convex BDPR succeeds for reasonable constants in sample complexity.

Blind Deconvolutional Phase Retrieval via Convex Programming

Ali Ahmed, Alireza Aghasi, Paul Hand

Funding provided in part by the National Science Foundation

December 5, 2018