Learning with SGD and Random Features

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Supervised learning

Given $(x_1, y_1), \ldots, (x_n, y_n)$

Learn a non-linear function $f : X \to Y$ e.g. $f(x) = w^\top \phi(x)$

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Goal: f provably accurate + computationally efficient

Learning with Stochastic Gradients & Random Features

$$w_{t+1} = w_t - \gamma \nabla L \left(w_t^\top \phi_M(x_t), y_t \right)$$

loss function

$$\phi_M(x) \rightarrow \begin{bmatrix} \sigma(\, s_1^\top \, x \,) \\ \vdots \\ \sigma(\, s_M^\top \, x \,) \end{bmatrix}$$

non-linear function

random sketching

SGD-RF with Mini Batching

$$w_{t+1} = w_t - \gamma \frac{1}{b} \sum_{i=b(t-1)+1}^{bt} \nabla L \left(w_t^\top \phi_M(x_{i_t}), y_{i_t} \right) \qquad t = 1, \ldots, T$$

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batchsize

nº random features

nº iterations

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Complexity: Time $\mathcal{O}(MbT)$ Space $\mathcal{O}(M)$

How to choose γ, b, M, T for optimal accuracy?

Our main result

Theorem(Carratino, Rudi, Rosasco 2018)

Let L be the squared loss

$$\underbrace{\mathbb{E}_{x,y} L(w_t^\top \phi_M(x), y) - \inf_w \mathbb{E}_{x,y} L(w^\top \phi_\infty(x), y)}_{\text{"Test error"}} \lesssim \frac{\gamma}{b} + \left(\frac{\gamma t}{M} + 1\right)\frac{\gamma t}{n} + \frac{1}{\gamma t} + \frac{1}{M}$$

Optimize w.r.t γ, b, M, T to get the best rate

Recipe

"Test error" $\lesssim \frac{1}{\sqrt{n}}$

Take $M = \sqrt{n}$

- $b = 1$ SGD single pass

$$\implies \gamma = \frac{1}{\sqrt{n}}$$

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$$\Longrightarrow \quad \gamma = \frac{1}{\sqrt{n}}$$

- $b = \sqrt{n}$ mini-batch single pass

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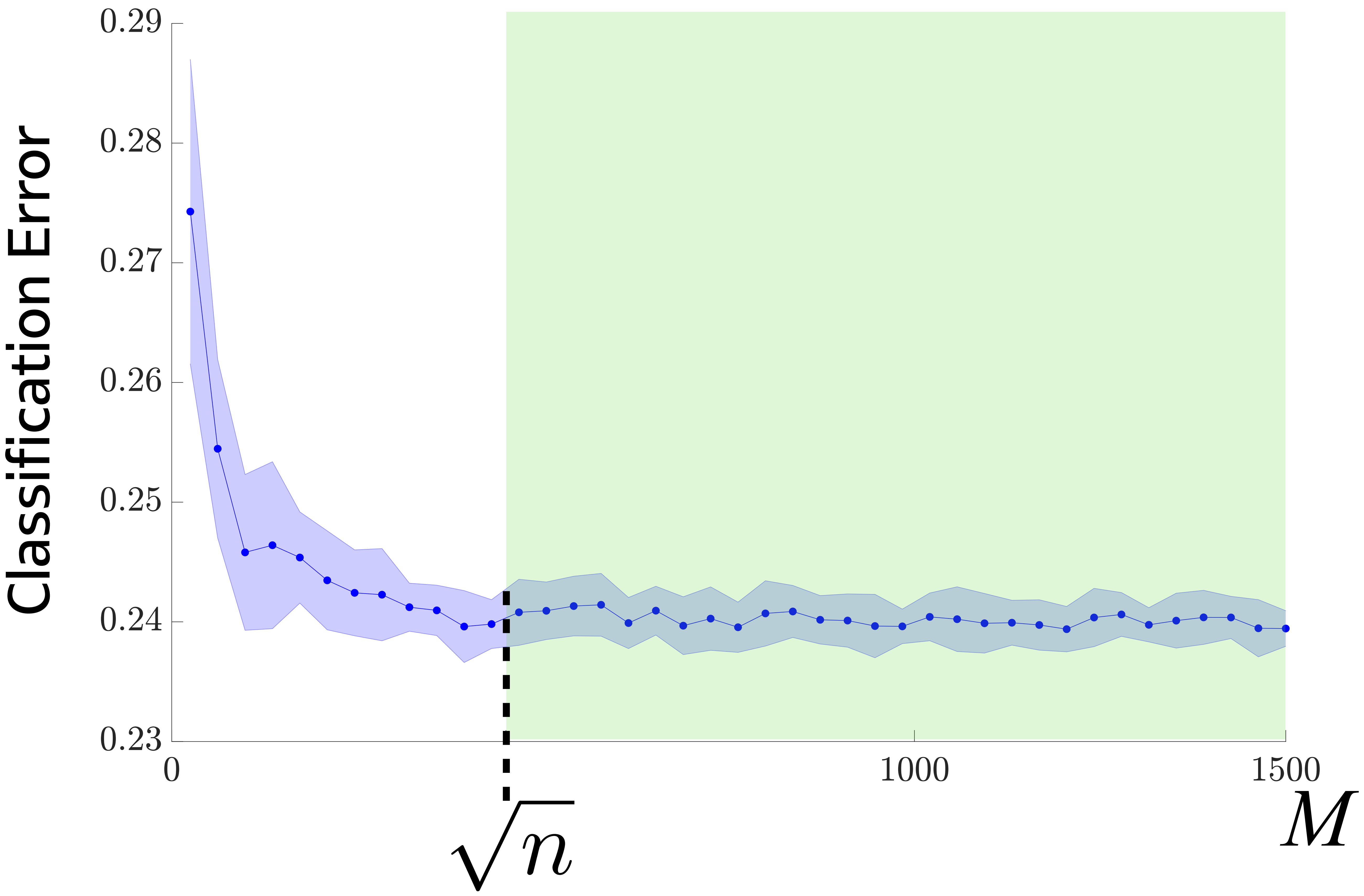
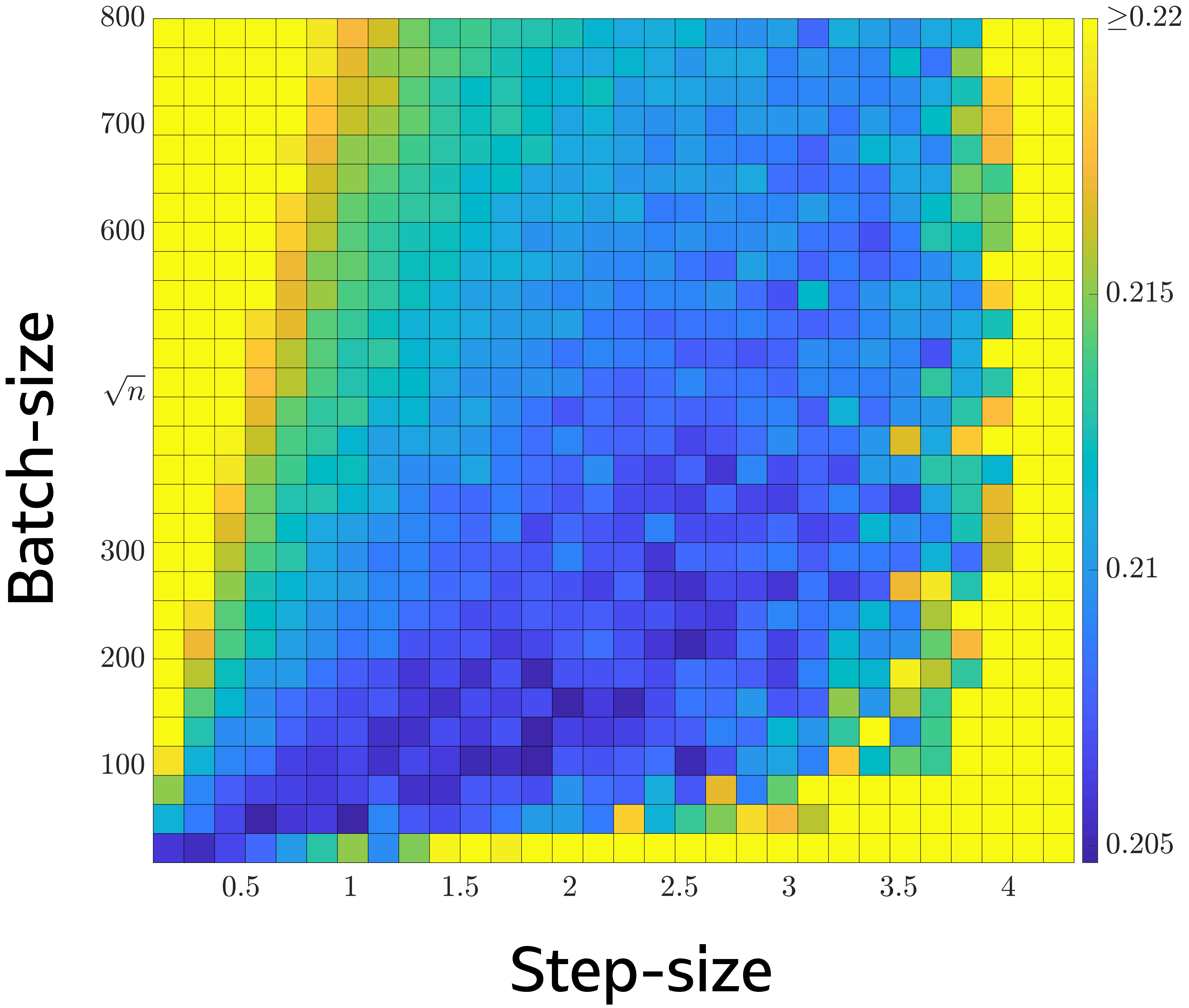
$$\Longrightarrow \gamma = 1$$

Complexity: Time $\mathcal{O}(n\sqrt{n})$ Space $\mathcal{O}(\sqrt{n})$

Practice validates theory

SUSY dataset $n \approx 10^6$





Contribution

SGD-RF leads to optimal accuracy with minimum computation

+ Faster rates

+ Decreasing step-size

Poster #127