Policy-Conditioned Uncertainty Sets for Robust Markov Decision Processes

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Advances in Neural Information Processing Systems 2018

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Why Robust MDPs?

- MDPs are powerful tools for modeling sequential decision making problems
- Transition probabilities are often **uncertain**
- Estimation errors can have detrimental effects on the resulting policies
- Unacceptable in applications involving high level of **risk**



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- Need solutions that are **robust** to this uncertainty

Problem

- **Robust MDPs** given sample trajectories from a reference policy $\widetilde{\pi}$
  - Build uncertainty sets Ξ containing the true parameters τ with high probability
  - Compute the optimal policy under the **worst-case** parameters in these sets

$$\max_{\pi} \min_{\tau \in \Xi} \ \mathbb{E}_{\tau,\pi} \left[\sum_{t=1}^{T-1} R(S_t, A_t, S_{t+1}) \right]$$

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- This problem is **NP-hard** in general [Mannor et al., 2012]
- **Rectangular** (independent) constraints [Nilim and El Ghaoui, 2005, Iyengar, 2005] provide tractability, but are too **conservative** and do not generalize

Non-Rectangular Uncertainty Sets via Marginal Features

- We consider **features** $\phi(s, a, s')$ to model the relationships between states and actions
- **Feature expectations** [Abbeel and Ng, 2004] to model the interaction of a policy π with the decision process

$$\boldsymbol{\kappa}_{\boldsymbol{\phi}}(\pi, \tau) = \mathbb{E}_{\tau,\pi}\left[\sum_{t=1}^{T-1} \phi(S_t, A_t, S_{t+1})\right]$$

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- Use feature expectations to define the **uncertainty sets**:

$$\Xi_{\widetilde{\pi}}^{\phi} = \left\{\tau : \boldsymbol{\kappa}_\phi(\widetilde{\pi}, \tau) = \widehat{\boldsymbol{\kappa}}\right\} \quad \text{or} \quad \widetilde{\Xi}_{\widetilde{\pi}}^{\phi} = \left\{\tau : \|\boldsymbol{\kappa}_\phi(\widetilde{\pi}, \tau) - \widehat{\boldsymbol{\kappa}}\| \leq \epsilon\right\}$$

Marginally-Constrained Robust MDPs

Constrained Optimization Problem

$$\max_{\pi} \min_{\tau \in \Xi^{\phi}_{\tilde{\pi}}} \mathbb{E}_{\tau,\pi}\left[\sum_{t=1}^{T-1} R(S_t, A_t, S_{t+1})\right]$$

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Properties

- Constrain whole trajectories rather than single states

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- Uncertainty sets are **policy-conditioned**

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- **Tractable** optimization

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Properties

- Constrain whole trajectories rather than single states
- Can **generalize** across the state space
- Uncertainty sets are **policy-conditioned**
- **Tractable** optimization
- Less conservative empirical performance than rectangular solutions

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